Evaluation of research and professional activity of research-oriented institutes of the Czech Academy of Sciences for the period 2015–2019

Final Report

Name of the Institute: Institute of Mathematics of the CAS, v. v. i.

Evaluated teams and their leaders:

1. Abstract Analysis (Wiesław Kubiś)
2. Algebra, Geometry and Mathematical Physics (Vojtěch Pravda)
3. Differential Equations and Theory of Integral (Robert Hakl)
4. Evolution Differential Equations (Šárka Nečasová)
5. Mathematical Logic and Theoretical Computer Science (Pavel Pudlák)
6. Numerical Analysis (Michal Křížek)

Introduction

The departments of the Institute of Mathematics are focused in basic mathematical research in Algebra, Geometry, Analysis (Functional and Abstract Analysis, Differential Equations) Numerical Analysis, Mathematical Physics, Logics and Theoretical Computer Science, strongly in development mathematical theory and methods in particular in general relativity, quantum mechanics, material sciences, solid mechanics, fluid dynamics, life sciences, computer sciences and computational sciences. In these areas, the Institute can build on a remarkable tradition that has produced an impressive list of outstanding, internationally leading scientists whose ideas and creative energy continue to serve as role models also to-day.

The motto "from tradition into the future" still has meaning in this institute and is a challenge that must be met by both the management and the staff.

The evaluation of this internationally highly regarded institute is based in particular on the direct assessment of the outputs submitted by the teams as the best contributions, as well as the information and material provided in the direct presentation of the institute and its teams.

The Commission acknowledges the very carefully prepared presentations, in particular the in-depth discussion of the implementations of the recommendations of the previous evaluations and the plans of the Institute for future developments.

Although all partners in the evaluation tried to compensate for the restrictions on direct communication with the members of the Institute caused by the Corona pandemic, this was only possible to a limited extent. The ad hoc organized discussion with the PhD students supervised at the Institute has proved to be very successful and useful.

Part A: Evaluation of the institute

Strengths:

The Institute of Mathematics continues to be a national and international top center in basic mathematical research and its transfer to important fields of application in science and society.

Its **great scientific tradition** has remained a defining factor for the Institute:

excellence and international competitiveness are emphasised as guiding principles in all ongoing activities, selection of new research directions and recruitment of new members.

The Institute has succeeded in ensuring both **stability and flexibility** in its scientific orientation while maintaining these principles. This became particularly evident in the structural and personnel changes in its scientific teams. So the Institute decided to close the team „Differential Equations and Theory and Integral“ and to integrate important and promising parts of the scientific program and of the staff in the teams „Evolution Differential Equations” and “Numerical Analysis” .

The presentations of the Institute and its teams and the very pertinent discussions reinforced the overall convincing impression of the Institute's **outstanding scientific achievements** in disciplinary and interdisciplinary research, which emerged from the direct, careful evaluation of the submitted outputs in Panel1.

The Institute is fortunate to have as members **several scientists, who are global leaders** in their sub-disciplines and who have had a seminal impact on research not only in their own field. Their important influence not only on the scientific program and outputs of the Institute, but also to its international reputation and attractiveness cannot be measured by standard indicators. They provide **important links to international scientific networks** and **are**

leaders of prestigious research projects, which are scientifically and financially important resources for Overall, the Institute has so far largely succeeded in shaping the age structure of the scientific staff while maintaining the high-quality criteria and satisfying the necessary demands.

Within the framework of an active guest programme, the Institute was able to welcome top scientists and young talents, some of them even for longer stays and more intensive cooperation. The number of researchers at the Institute grew from about 70 to about 90. The growth comes partly from attracting new members from other countries, which is an important indicator of the success of the Institute.

The Institute has a long **tradition of close cooperation with universities in both research and teaching.** It is particularly evident in the cooperation with Charles University within the frame-work of the Necas Center, in which the training of young scientists plays a special role, and more recently with the Einstein Center.

The infrastructure and service units of the Institute form a valuable basis for a research center in which national and international cooperation and the exchange of scientists play an important role. Its mathematical library provides important services for the Czech Republic.

Weaknesses:

Despite the good relations with universities and substantial contributions to education, in which joint workshops and schools on attractive topics are very attractive nationally and internationally, there is an urgent need for cooperation of Master and PhD students. In mathematics and computer science, it is essential to get access to young talents very early and to involve them in research and teaching at an early stage. The institute directly funds about 10 PhD students, but members supervise a total of 30 PhD students. This figure shows improvements since the evaluation 5 years ago. However, there are still deficits that have to be addressed.

One important reason for the deficit is on the one hand structural in the education system, but also in the way education and research are financed in the Czech Republic. In order to achieve the necessary changes, practicable cooperation agreements must be made, whereby one should be guided by the experiences in other European countries.

The structure, the amount and the stability of their funding are causing crucial difficulties for most Institutes of CAS and reasons for permanent complaints.

Opportunities:

The Institute makes intensive and successful use of national and international funding programmes and cooperations with industry and business to finance its projects and uses them to ensure both the quality and the ongoing necessary renewal of staff. It is directly involved in AV21 providing in particular its expertise in mathematical modelling, analysis and simulations of model systems.

After its reduction to 5 teams, the Institute has very good potential for analysing new topics and strengthening internal bridges between internal teams.

There is also an opportunity to better link central disciplines together with the other institutes or to find a suitable coordinated task sharing for the benefit of all. This applies in particular to the areas of Logic, Numerics, Statistics and Stochastics.

The mathematical institute has a special role to play here, as it is strongly oriented towards basic mathematical research.

The institute also offers the best conditions for a school for master and doctoral students jointly supported with universities, also integrating this education and research projects in joint centers like Necas Center.

Threats:

The aforementioned weaknesses threaten the HR Policy and the ambitious goal of the institute not only to maintain the high level of disciplinary and interdisciplinary research and its transfer, but also of making it a significant European research centre in mathematics.

The Commission had a detailed discussion with the Institute's management about the HR policy and strategy in planning the orientation of future research about quality assurance measures, which generally apply to all the institutions it assesses. It sees a need for discussion above all at the CAS level.

Main criterion: 1. Quality of results (H1.1-H1.5)

H1.1	Quality of selected outputs of Phase I

The procedure used in Panel I results in a classification of the quality of results in the disciplines of mathematics and computer science, the significance of which is to be rated higher than assessments based on purely bibliometric data, which of course should be taken into account. For the assessment of results, the expertise of experts in the specific subject area as well as of scientists with a broader overview of the discipline is necessary. The submitted publication must be read and evaluated. The Institute of Mathematics was highly successful in the rating of the 115 outputs it selected, as the following distribution of ratings shows

1	2	3	4	1 or 2
28,7%	53,9%	16,5%	0,9%	82,6%

1 „*world leading*“, 2 „*internationally excellent*“, 3 „*recognized internationally*“ *4* „*recognized acceptable*”

This assessment shows that the institute in research is internationally in the top league and with a noteworthy share of research even globally.

It should be emphasised that the submitted outputs of “*Mathematical Logic and Theoretical Computer Science*” received the rating with *5*2,0% world leading and 44,0% internationally excellent.

Also, the team “*Differential Equations and Theory of Integral”,* which was decided not to be continued, could achieve a very positive evaluation for its contributions in the period 2015-2019, justifying the integration of remaining staff in other teams.

H1.2	Contribution of workers on the outputs reached

In the review of the submitted outputs in Panel I, care was taken with several authors to ensure that appropriate participation of the Institute's staff could be assumed This is possible, by taking into account among other things the required expertise of the participating authors and their previous work.

H1.3	Quality of all outputs and results

An in-depth review, as was carried out in Panel 1 for a comparatively large number of outputs submitted by the teams, is too costly for the large numbers at the Institute. Data submitted bibliometrically were used to check the quality of the outputs in samples in the individual teams of the Institute. This revealed that the ratings obtained in Panel I became a shift since the evaluated outputs were preselected by the teams. Despite this expected effect, the result in Panel 1 retains significance for the classification of the quality of the institute and its teams.

H1.4	The most valuable discoveries and findings in the fields, their importance for the field

- W. Kubis: Injective objects and retracts of Fraisse limits. Forum Mathematicum. 2015
- Hervik, V. Pravda, A. Pravdová: Universal spacetimes in four dimensions. Journal of High Energy Physics, 2017
- G. A. Monteiro, A. Slavík, M. Tvrdý: Kurzweil-Stieltjes Integral: Theory and Applications. 1. New Jersey: World Scientific, 2019. Series in Real Analysis, Volume 15.
- D. Breit, E. Feireisl, M. Hofmanová: Stochastically Forced Compressible Fluid Flows, De Gruyter Series in Applied and Numerical, Mathematics, Volume 3, 2018
- Hrubes and P. Pudlak: Random formulas, monotone circuits, and interpolation. In: 2017 58th IEEE Annual Symposium on Foundations of Computer Science (FOCS). New York: IEEE, 2017.
- P. Allen, J. Böttcher, J. Hladký, D. Piguet: Packing degenerate graphs. Advances in Mathematics. 2019
- E. Carson, M. Rozložník, Z. Strakoš, P. Tichý: The numerical stability analysis of pipelined conjugate gradient methods: historical context and methodology. SIAM Journal on Scientific Computing, 2018.

H1.5	Contribution of the participation of the authors in large collaborations

Collaborations in large groups are rather rare in research at the Institute. With a larger number of authors, the Institute's share and the quality of its participating scientists is usually such that its contribution to the results can be considered substantial.

Main criterion: 2. Societal relevance (H2.1-H2.5)

H2.1	Societal relevance of outputs and results pursuant to CAS and institute mission

The Institute's contributions lie mainly in basic research in central and cutting-edge research areas in mathematics and computer science and their applications, initially in the sciences, which are increasingly dependent on the use of mathematical and computational methods. In close cooperation with its CAS partners Institute of Computer Science and Institute of Information Theory and Automation the Institute is developing theory and methods that are basic for Computational Sciences. These are critical enabling disciplines underpinning advanced research and development in all fields of natural sciences, engineering, medicine, social sciences and humanities.

Also, industrial innovation is increasingly based on the results and techniques of scientific research, both underpinned and driven by mathematics and computer science. Especially in the current situation, where industry has to respond to a list of critical challenges, it is necessary and possible through industry-relevant basic research in mathematics and computer Science. In particular, the Evolution Differential Equations, Numerical Analysis, Mathematical Logic and Theoretical Computer Science teams offer research results and appropriate expertise for substantial cooperation with industry. The potential is already partially exploited, but still offers opportunities for expansion, both for the benefit of the industrial partner and for industry-related basic research.

H2.2	System functionality for knowledge transfer into practise, its usefulness for society. The impact of the institute´s activity on proper practice in society in the area of social sciences and humanities

It makes little sense for institutes that are so focused on basic research, which is strongly application-oriented but does not directly conduct applied research, to operate their own transfer facilities. In contrast, it makes a lot of sense to be involved in the transfer of

innovative research results and the development of new, practically relevant scientific problems for basic research. The Institute does this in particular by actively participating in corresponding national and international platforms.

The potential to contribute to new projects in computational social science or computational humanities exists in principle at the Institute. Currently, methods of computational topology in network analysis are becoming increasingly important in the social and human sciences.

H2.3	Relation to practice

The interpretation of "research for practice" as the development of marketable products is too narrow, especially from the perspective of industry-relevant basic research.

Mathematical modelling and simulation can help to transform the problem of practice into a mathematical problem that needs to be solved using mathematical and computational methods. The investigations carried out on a "digital twin" of the real situation provide valuable information for the design of a practical solution, which is then better realised by the practice partners. Seen in this light, the Institute has good potential to successfully participate directly in the solution of practical problems, and it should make use of this potential.

H2.4	Participation in AV21 strategy

The Institute is involved in the AV21 programme “Hopes and Risks of the Digital Era” together with the Institute of Information Theory and Automation, Institute of Computer Science. The role of the Institute is to develop mathematical models for engineering applications within the sub-programme “Mathematical modelling as an efficient tool for the control of complex processes”.

The results achieved so far have already been applied in an existing cooperation of the institute with the innovation centre of an internationally renowned producer of compact loaders and excavators.

The Commission acknowledges the Institute's substantial participation in Initiative AV21. This Initiative is also seen as very positive, as it aims to use a synergy of competences at the participating CAS institutes to develop mathematical and computational concepts and methods required for real-world applications and to promote the direct transfer into relevant applications.

H2.5	Cooperation with regions of the Czech Republic

The Institute of Mathematics does not have an official cooperation with the regions. How its cooperations with regional universities and its services have a very positive influence on the development in the regions.

Further criterion: 1. Position in international and national context (D1.1-D1.3)

D1.1	Comparison of the teams and the institute with similar international and national institutes

The Institute of Mathematics continues to be a national and international top center in basic mathematical research and its transfer to important fields of application in science and society.

The international recognition varies among the teams of the institute. Clearly, the teams Evolution Differential Equations and Mathematical Logic and Theoretical Computer Science are highly recognized while other teams, e.g. the team Numerical Analysis, have the potential to reach this point.

D1.2	Scope and quality of international and national cooperation and the role of the institute in such cooperation; engagement in broad international cooperation

It represents an important node in international networks, e.g., in the network in EU-MATHS-IN for strengthening mathematics in industry, in zbMATH Open, one of the two leading global mathematical databases.

D1.3	Participation of the workers in scientific community activities (organizing of conferences and workshops, invited lectures, awards)

The members of the institute are extremely active in serving the scientific community. They regularly (co)-organize large, renowned international conferences, many workshops focused on special topics and schools for PhD students and young scientists. Many members of the institute are regularly invited to give plenary talks at conferences as well as further invited lectures. This proves the international reputation of the institute and its members. The institute is active in proposing its members for scientific awards and prices and is quite successful with this policy.

Further criterion: 2. Vitality, sustainability and strategy (D2.1-D2.9)

D2.1	Direction in line with the perspective of the planned research directions

The institute guidance principles are scientific excellence in all aspects. It sees its role in providing a stimulating environment for fundamental research in mathematics including the freedom of individual researchers to choose their research subjects. The organization in teams working on well-defined topics ensures that the overall research topics are not too much diversified. They fit well to the general aim of the institute.

D2.2	Assessment of the previous research objectives and their achievement

According to the team reports most previous research objectives were reached. In case a major part of the objectives was not reached reasonable explanations were given (e.g. the movement of a researcher responsible for that topic to another place). Moreover, new modern research topics were identified. These new trends are supported by a newly introduced visitors' programme and by opening several new open topic positions for researchers.

D2.3	Assessment of implementation of recommendations from past evaluation

The institute was evaluated in the last evaluation by two different commissions from mathematics and from computer and information sciences. These were:

1. Focusing the research program of the team to central topics

   The teams emm to be on a good way.

2. Identifying and selecting new research directions

   Due to changes in the personnel, new research directions were opened in some of the teams. The DETI team will be closed and some of its former team members will move to the EDE team. However, from the documents an overall strategy to identify new topics is not visible.

3. Allocating existing resources following defined priorities

   The top priority of the institute is excellent research and resources are distributed among the teams following this guiding principle. This might have the effect that smaller teams have a smaller chance to hire additional research staff.

4. Research program and structure of the DETI team have to be reconsidered...

   The DETI team will be closed and its remaining researchers will be joined with the EDE team. This makes sense since some topics of the DETI team fit well to the research on solids performed in a subgroup of EDE.

5. The staff necessary to represent Numerical Analysis appropriately should be assigned to the NA team

   The Numerical Analysis group was strengthened by hiring an experienced researcher with topics in numerical linear algebra. In this way also the collaboration with the Institute of Computer Science was deepened in an unexpected way. The team now has a good structure for its future research activities,

6. The Commission is recommending a consultation of the Academy and the Institute on one side and the Universities and the Faculties on the other side with the aim, to improve the cooperation in particular in the education of students on all levels.

   Members of the institute teach on all levels at Universities. However, it is difficult for the Institute to have access to talented young students for obtaining PhD students. This seems to be a structural problem. First steps were done by participating in joint PhD programmes (MathInHPC, Prague doctoral school (in physics)). These activities should be extended to all subjects and the Institute is encouraged to fight for an extension of the aforementioned programmes.

7. Support for the Necas Center

   The institute is active within the Necas Center for instance by launching the Necas Center Series of publications from the members of the Center. The Institute is encouraged to actively use the platform of the Necas Center for a structured collaboration with other mathematical groups in Prague. For new researchers coming to Prague the Necas Center is a perfect platform to come into contact with the mathematical groups in Prague.

D2.4	Success in receiving grants

The institute was successful in grant applications, the main contributions coming from two ERC grants. Most grants by number were received from the GACR. The GACR Junior grants seem to be unevenly distributed among the teams. Young team members should be strongly encouraged and supported by experienced scientists to apply for junior grants.

The preparation of project proposals is strongly assisted by a project manager who substantially contributes to the success of the Institute in this field.

D2.5	Adequacy of instrumental equipment

The Institute of Mathematics provides a well-equipped library including the access to e-journals and databases, which is sufficient since the majority of the research belongs to theoretical science. For HPC, sufficient computing capacity could not be easily provided by IT4I, the Computing Centre of Ostrava.

D2.6	Effectiveness of management

During the evaluation the institute‘s director and the deputy director gave a very competent and well informed impression about all aspects of the institute.

According to the institute report, administrative positions are filled by well-trained staff and continued training for this group seems to be an important point for the director of the institute.

The managing team of the institute includes a project manager who is in charge of the administrative aspects of large projects. This reduces the administrative loads that come along with project funds.

D2.7	Assessment of professional structure, development strategy and the strategy of keeping best scientists, age structure, career and qualification growth

The institute has a clear hiring and career policy. New positions in general are announced in international open-topic calls in order to attract top scientists and also to stay open for new research directions. The HR policy part of the institute‘s report lists several reasonable measures to promote scientists. One building block are the regular personal evaluations in line with the CAS rules carried out by the Recruitment and Evaluation Committee. However due to the limited amount of the basic funds provided by the Academy the flexibility for active personnel recruiting strategy is restricted. The institute is successful in allocating additional funds. But these do not allow the institute to develop a broad strategy for long term perspectives for promising scientists.

The institute succeeded in receiving the funding for implementing a professional HR strategy.

An important point is the active approach to researchers to apply for the DSc degree.

The age-structure improved with several newly appointed scientists with an age between 30 to 45 years. This group of younger scientists provides a good pool of future team leaders.

In the recent years the level of internationality has increased, which shows that the institute is internationally highly attractive as a place to build a research career in mathematics.

D2.8	Creating work-life balance conditions, assessment of approach towards possible gender issues

The institute provides support for the cultural, recreational and sporting activities of its employees, for language courses and also runs a combined common room and tearoom for informal meetings and relaxation. However, during the pandemic some PhD students miss spontaneous networking opportunities with members of the institute (and not only their own group) such as a (weekly) virtual tea time, for instance.

A big plus of the institute is the ability to offer new and in particular to foreign employees temporary apartment accommodation in the institute‘s building which makes the search for a long-term accommodation less stressful.

Researchers with family care duties may organize their working hours according to their individual needs. In the discussion subsequent to his presentation the director confirmed that time out for family care is taken into account in internal evaluation procedures. The institute has no special measures concerning gender balance issues.

D2.9	Relation of the institute with regard to the integration, development and sustainability of the research centre funded by the National Programme of Sustainability II.

Not relevant.

Further criterion: 3. Cooperation with universities and participation in education (D3.1-D3.6)

D3.1	Scope of cooperation with universities on national and international level

The Institute of Mathematics at CAS occupies an important place in the research landscape of Czech Republic and its regions. It maintains very intensive contacts, especially with Charles University and the Technical University in Prague, with the University of West

Bohemia in Plzeň, with Masaryk University in Brno, but has also strengthened its relations with Ostrava, especially with the IT Supercomputer Centre there.

As a special example in this context we mention the doctoral school MATH-IN-HPC.EDU (with TU Ostrava, Charles University). The Commission encourages the institute to fight for an extension of this school beyond 2022.

On the international level the institute has close ties to several universities and the members of the institute regularly teach short courses all over Europe but also in Botswana, China, Korea and USA, for instance.

D3.2	Effectiveness of joint research centres

The Institute is a main partner of the universities in two joint research centres promoting joint research and education of the young generation:

- Nečas Center for Mathematical Modelling,
- Centre for Discrete Mathematics, Theoretical Computer Science and Applications - (DIMATIA).

During the evaluated period it also was a member of the following centers, which only were funded for limited period:

- Albert Einstein Centre for Gravitation and Astrophysics,
- Centre of Excellence Institute of Theoretical Informatics,
- Eduard Čech Institute for Algebra, Geometry and Physics.

The Commission supports strongly to continue activities of the institute especially in Nečas Center, which is a model for successful cooperation between CAS institutes and universities in the region, in particular in the integration of research and teaching.

D3.3	Success rate in supervision of PhD students

Non-university institutions are allowed to train PhD students only if the institution is accredited to participate in the doctoral programme. At the moment, the Institute of Mathematics has such agreements only with the Charles University and the University of West Bohemia in Pilsen. Hence, the official number of students (10 in the period under evaluation) differs from the number of students that is (co-)supervised by members from the institute (58, 31 defended thesis within 2015–2019). More than half of the students who finished their PhD in 2015–2019 got positions in academia, in Czech Republic and abroad. Only four stopped their study without a degree. This shows that after finishing their PhD the students are internationally competitive.

However, looking at the total number of PhD students there is space for improvement. Many groups in the institute complained that due to the general structures it is difficult to attract talented PhD students to the Institute of Mathematics. This is a general problem of many institutes of CAS and should be tackled on a higher level.

D3.4	Participation of PhD students in the outputs

In all groups PhD students contribute to the scientific output of the teams and are co-authors for publications. Some outstanding students reach up to ten (co)-authored publications.

D3.5	Participation of the institute in master or bachelor studies

Members of the institute regularly give courses on the bachelor and master level. They also supervise theses. The total number in particular concerning the master theses seems to be a bit low (5 bachelor, 8 master thesis in 5 years) since from this group one would expect to attract PhD students. However, this might also be a general structural problem.

D3.6	Assessment of cooperation intensity with universities in the form of teaching
Members of the institute teach on all levels (bachelor, master, doctoral) at several universities in the Czech Republic. They also supervise bachelor and master theses. This is an extremely important activity in order to attract students to the institute. It is important to continue with these activities on a very high level.	

Further criterion: 4. Outreach activities (D4.1-D4.3)

D4.1	Sufficiency of media strategy and activities in the area of research popularisation
The institute regularly contributes with lectures, presentations and excursions to several public events, like for instance the Week of Science and Technology and the Science Fair and reaches a broad audience with these activities. It further organizes an annual lecture series open for a broader audience with mathematical background and with internationally renowned speakers. It supports the Mathematical Olympiad both with people and financially.	
D4.2	**Publishing activities and its quality**
One book – winning a prize for scientific literature -- as well as two poster series for a broader audience were published, several public lectures were recorded and are accessible through the institute‘s web-page. Games and computer programs related to research topics of the institute were developed and received attention in the media.	
D4.3	**Participation in professional organisations in the area of research and development**
In addition to many workshops and schools tailored to specific scientific topics the members of the institute regularly organize large conferences within international conference series. They are members in several international boards, scientific councils and committees (e.g. ERC Synergy Grant Panel), have leading roles in editorial boards and are referees for grant agencies. To summarize, the institute is well represented in several different committees on the international as well as on the national level.	

Recommendations:

The Commission recognizes the high international scientific standards in the Institute's central thematic areas. Several leading top researchers play a central role in this achievement. In view of the current age structure of the scientific staff, it is recommended that in future appointments, successors be found well in advance, especially for these leading positions. Current need for action is seen in the Logics and Computer Science team.

There is potential for increased scientific cooperation both between teams in the Institute and with teams in other CAS institutes. This applies in particular to the teams for Evolution Differential Equations (EDE), Numerical Analysis (NA) and Algebra, Geometry and Mathematical Physics (AGMP). Examples of topics for improved cooperations are

- EDE – NU: analysis and numerics of fluid-structure interactions, fluid dynamics with stochastic forcing, differential-functional equations;
- EDE, NU – AGMP: analysis and numerics of Yang-Mills-equations;
- EDE, NU – Institute for Information Theory and Automation: Decision Making Theory (DMT), Stochastic Informatics (SI);
- EDE, NU – (SI) differential equations with stochastic forcings,
- EDE, NU – (DMT) analysis and numerics in mechanics of solids.

The research area Numerical Linear Algebra is no longer in the program of the Institute of Computer Sciences. It is strongly recommended to continue this highly topical field within the Numerical Analysis Team of the Institute of Mathematics as an important contribution to numerical algorithms for complex, high-dimensional systems and to High Performance Computing.

The Commission supports the initiative to start a focus on C*-Algebra linked with the team Abstract Analysis - see the team evaluation and recommendations.

Other comments of the commission:

We have recommended that all teams are left at the current level of support except for Mathematical Logic and Theoretical Computer Science, were we recommend increased support to recruit a replacement for Pavel Pudlak as leader of the group. We appreciate that other groups will face the same problem of replacing a scientific leader and these should be dealt with on a piecemeal basis as they arise. We also appreciate that the Numerical Analysis: Team has been depleted because both the Director and Deputy Director of the Institute have been recruited from within this group. We do not have scientific grounds for recommending any change of level of support since this is an internal management issue. We do though suggest that the Institute should monitor this issue carefully and take it into account in its future planning. We also note that CAS has lost the important field of Numerical Linear Algebra, since the strong team, led by Z. Strakoš has left CAS to join the Department of Mathematics at Charles University. If CAS considers restarting this activity, then the Numerical Analysis team would form a good nucleus for it. Finally, we note that the Differential Equations and Theory of Integral (DETI) Team has been dissolved and its members are being redistributed. We therefore make a null return under the recommendation of change of level of support. However, we do recommend that resource adjustments made to the Teams into which the former members of DETI are transferred.

Part B: Evaluation of teams

1. Abstract Analysis

Strengths:

Analysis in Prague is traditionally a strong subject, and the present group managed very well to continue this trend, with a remarkable opening to international students and researchers. The team shows a nice balance between thematic unity and diversity of the contributions.

Weaknesses:

No obvious weakness in our opinion, beyond the global challenge of obtaining grants on a regular basis, as is necessary today for the continuity of fundamental research.

Opportunities:

Prague is now an attractive city at the centre of Europe, and the Mathematics Institute runs many collaborations with neighbouring countries. This should be an opportunity for recruiting internationally. It has already been done in recent years, and this excellent strategy should be encouraged.

Threats:

No obvious threats. In our opinion, the team manages well to avoid a classical problem in fundamental mathematics, which is to focus on old unsolved problems which do not attract much attention any more, and its fields of research are consistently renewed.

Main criterion: 1. Quality of results (H1.1-H1.5)

H1.1	Quality of selected outputs of Phase I
The team achieved the following distribution of ratings	
1	2	3	1 or 2
17,4%	52,2%	30,4%	69,6%
1 *„world leading“*, 2 *„internationally excellent“*, 3 *„recognized internationally“*	
Excellent output, showing that the team has an indisputable international level.	
H1.2	**Contribution of workers on the outputs reached**
All papers have major contributions by team members. (In fundamental mathematics, it is unusual to specify who did what in paper with more than one author.)	
H1.3	**Quality of all outputs and results**
The team is productive in publications in pure mathematics journals, for which bibliometric data such as impact factors and citations are misleading.	
H1.4	**The most valuable discoveries and findings in the fields, their importance for the field**
It is pointless to summarize 145 articles in a few lines, but we single out a few contributions by way of illustration: The deep work of W. Kubis on Fraïssé limits and generic structures,	

Jan Kolar's work in geometric measure theory and on extension of Baire one functions and smooth functions, O. Kurka's contribution to descriptive set theory and Banach spaces, M. Fabian on non-separable Banach spaces and P. Hajek who clearly is the world leader on smoothness in the infinite-dimensional setting, and the work of V. Müller on linear dynamics and operators on the Hilbert space. In a joint work with M. Malicki, M. Doucha investigated generic representations (in the Baire category sense) of countable groups, and solved in particular a problem stated by A. Kechris and C. Rosendal. But this list is only a sample of the contribution of this dynamic team to mathematical analysis, and we refer the reader to the comprehensive report provided by the team leader.

H1.5	**Contribution of the participation of the authors in large collaborations**
There are no large collaborations in the sense of category "D".	

Main criterion: 2. Societal relevance (H2.1-H2.5)

H2.1	**Societal relevance of outputs and results pursuant to CAS and institute mission**
The research produced in the Institute of Mathematics is by nature fundamental research, but the main topics are central and relate to other parts of mathematics. In other words, they are basic and somehow universal. Immediate societal relevance mainly concerns connections to computer science or applicable mathematics, e.g. via partial differential equations.	
H2.2	**System functionality for knowledge transfer into practise, its usefulness for society. The impact of the team´s activity on proper practice in society in the area of social sciences and humanities**
Many parts of fundamental mathematics have no *direct* technical impact on social sciences and humanities. However, the rigorous procedures mathematicians use for showing their results deserve to be better known at a time when asserting without proof is fashionable. In this respect mathematicians may have an immediate impact on society.	
H2.3	**Relation to practice**
Mathematics is used in every industry, including banks and the stock exchange. The pace of mathematics is rather slow and this is unavoidable. Some of the research produced today might be used much later, in a rather unexpected way.	
H2.4	**Participation in AV21 strategy**
The AV21 strategy is an independent body, which interacts with most of the Institutes of the CAS, although its relevance to Abstract Analysis is probably small.	
H2.5	**Cooperation with regions of the Czech Republic**
There is no official cooperation with the regions.	

Further criterion: 1. Position in international and national context (D1.1-D1.3)

D1.1	**Comparison of the team with similar international and national institutes**
Many members of the team operate at the highest international level in their discipline. Researchers and students are fully international, and actually the team is now very open to	

recruiting foreign students and researchers. Moreover, the Abstract Analysis team gathers a very dynamic group of young researchers. In our opinion, the « Prague School » is now among the top five teams in Europe in functional analysis.

D1.2	Scope and quality of international and national cooperation and the role of the team in such cooperation; engagement in broad international cooperation
Members of the team have many active international collaborations with members of established research institutes and universities, in neighbouring countries, Asia and North America.	

D1.3	Participation of the workers in scientific community activities (organizing of conferences and workshops, invited lectures, awards)
Members of the team are active as editors or associate editors of international journals and the mature members of the team are fully doing their job in this matter.	

Further criterion: 2. Vitality, sustainability and strategy (D2.1-D2.9)

D2.1	Direction in line with the perspective of the planned research directions
Progress in fundamental science is obviously difficult to plan ahead, both in direction and in schedule. However, this team clearly has the ability to renew its centers of interest. The project of a new direction of research in C*-algebras and non-commutative geometry is a good idea which must be supported.	

D2.2	Assessment of the previous research objectives and their achievement
Planning research in fundamental mathematics over a period of five years can only be very global in nature. But the team did make progress in each of the areas on which it had planned to focus its attention (universal homogeneity, Bergman spaces, Banach space theory) and started a new focus area, in C*-algebras.	

D2.3	Assessment of implementation of recommendations from past evaluation
The team was invited to fill the gaps concerning the younger generation and it succeeded perfectly well in doing so. The Prague team of functional analysis is presently one of the most dynamic in Europe (and beyond) and CAS can get full credit for this. Also, the status of general topology, which is no longer a trendy topic, had been mentioned in the past evaluation: the team certainly agreed with this fact and « core topology » is clearly not in its present focus.	

D2.4	Success in receiving grants
At present, the team uses the prestigious EXPRO grant (led by the team leader W. Kubis), three junior grants from the Czech Science Foundation (for M. Doucha, Th. Kania and K. Strung) and an international research grant with Austria.	

D2.5	Adequacy of instrumental equipment
Not relevant.	

D2.6	Effectiveness of management
The team leader W. Kubis clearly has a deep understanding of what is done in his team, and moreover he has the ability of attracting funding and PhD students. Much of the management is done at the institute level. It seems that things are run smoothly.	

D2.7	Assessment of professional structure, development strategy and the strategy of keeping best scientists, age structure, career and qualification growth
The team is well structured, covering different but related areas. The age distribution is good, with most of the researchers between 30 and 40. But it should be stressed that the senior generation is still very active and efficient.	
D2.8	**Creating work-life balance conditions, assessment of approach towards possible gender issues**
Seems to be dealt with adequately at the Institute level.	
D2.9	**Relation of the team with regard to the integration, development and sustainability of the research centre funded by the National Programme of Sustainability II.**
Not relevant.	

Further criterion: 3. Cooperation with universities and participation in education (D3.1-D3.6)

D3.1	Scope of cooperation with universities on national and international level
Cooperation in research is mostly at an individual level, but it is widespread and fully international.	
D3.2	**Effectiveness of joint research centres**
Most of the cross-institutional cooperation concerns Universities, which is natural in fundamental mathematics.	
D3.3	**Success rate in supervision of PhD students**
In the period under evaluation, 8 PhD students defended their theses. Wieslaw Kubis and Petr Hajek are particularly successful as international advisors.	
D3.4	**Participation of PhD students in the outputs**
The PhD students actively contributed to the research output, as authors or co-authors. As usual in mathematics, most of the articles are signed by very few authors, and only by people who significantly contributed to the work.	
D3.5	**Participation of the team in master or bachelor studies**
Members of the team gave a significant number of lectures in various universities, but it would certainly be good to increase this pedagogical activity.	
D3.6	**Assessment of cooperation intensity with universities in the form of teaching**
It is advisable to increase the teaching done, at the graduate level, in Universities. But of course, it might be easier said than done since Universities are not always keen to share their graduate students. It would be good to aim at enlarging the number of master thesis supervisions, with the goal of attracting more PhD students.	

Further criterion: 4. Outreach activities (D4.1-D4.3)

D4.1	Sufficiency of media strategy and activities in the area of research popularisation
The team participated in various important activities in the direction of diffusion of mathematics, and is clearly ready to do more in this direction. A related question is whether local High Schools are willing to participate to « science fairs » and to invite mathematicians for delivering talks to their students.	
D4.2	**Publishing activities and its quality**
In the context of outreach, the team contributed to a European digital mathematics library.	
D4.3	**Participation in professional organisations in the area of research and development**
No information seems to have been provided on this point.	

Other comments of the commission:

2. Algebra, Geometry and Mathematical Physics

Strengths:

The team has produced excellent research in several directions, and was able to attract several foreign new members at the postdoc and researcher level. The team grew considerably in size over the evaluation period (from 8 to 15 FT), partly because of Markl's Praemium Academiae grant, but also because of success in other granting schemes. Several members of the team interact successfully with researchers at Charles University, e.g. through the Einstein Centre for Gravitation and Astrophysics.

Weaknesses:

The team lacks a bit of coherence, and gives the impression that the relativity group publishing in physics journals stands apart from the others who work in core subjects in algebra and geometry. The team has been less active than some other groups in teaching courses at the university (11 master courses in the evaluation period), and perhaps related to this has supervised a small number of PhD students.

Opportunities:

The recent participation in the Prague Doctoral programme has resulted in two new PhD students, and forms an example of a direction in which the team could become more active.

Threats:

The team lacks cohesion as said, and depends rather heavily on Markl's grant. The team might wish to focus on a strategy for the period after this grant expires.

Main criterion: 1. Quality of results (H1.1-H1.5)

H1.1	Quality of selected outputs of Phase I
The team achieved the following distribution of ratings	
1	2	3	1 or 2
27,3%	50,0%	18,2%	77,3%
1 *„world leading“*, 2 *„internationally excellent“*, 3 *„recognized internationally“*	
The Team selected 22 outputs of which 17 were classified as lying in categories 1 and 2. The team published in solid state physics journals such as J High Energy Physics and very good general mathematics journals such as Advances in Mathematics.	
H1.2	**Contribution of workers on the outputs reached**
The members of the team played an essential role in all publications. The fractional count in categories 1 and 2 is 73 %.	
H1.3	**Quality of all outputs and results**
The team has been quite productive over the evaluation period, and published very good to excellent results. A fair number of publications not selected for Phase I were published in categories 1 and 2 journals.	
H1.4	**The most valuable discoveries and findings in the fields, their importance for the field**

The team achieved important results on the existence of universal spacetimes (Pravda et al, J High Energy Phys, a.o.), and made important contributions to the foundations of Yang-Mills theory (Schreiber). Furthermore, it developed a new and fruitful notion of operadic category which will lead to much further research (Markl, Advances in Mathematics), and made important progress in the classification of tilting modules in representation theory (Positselski) and t-structures (Hrbek). Hong Van Le published a book in the prestigious Ergebnisse series.

H1.5	Contribution of the participation of the authors in large collaborations
There are no large collaborations in the sense of category “D”. The team has many and widespread international collaborations on an individual basis.	

Main criterion: 2. Societal relevance (H2.1-H2.5)

H2.1	Societal relevance of outputs and results pursuant to CAS and institute mission
The team has made many important advances in fundamental science which have a direct bearing on other more applied parts of physics and mathematics such as statistics. As usual for fundamental science, applications to society are indirect.	
H2.2	**System functionality for knowledge transfer into practise, its usefulness for society. The impact of the team´s activity on proper practice in society in the area of social sciences and humanities**
Not relevant.	
H2.3	**Relation to practice**
There is no direct relation to practice.	
H2.4	**Participation in AV21 strategy**
See remarks at the institute level.	
H2.5	**Cooperation with regions of the Czech Republic**
There is no official cooperation with the regions.	

Further criterion: 1. Position in international and national context (D1.1-D1.3)

D1.1	Comparison of the team with similar international and national institutes
The team functions at the same level as some of the top institutes internationally, as witnessed by the collaborations below.	
D1.2	**Scope and quality of international and national cooperation and the role of the team in such cooperation; engagement in broad international cooperation**
The theoretical physicists have a strong collaboration with a group in Stavanger, for example. At an individual level, the researchers in algebra and geometry have widespread international collaborations (Sydney, Purdue, Minnesota, Regensburg, Padova, Toulouse, and many other universities.)	

D1.3	Participation of the workers in scientific community activities (organizing of conferences and workshops, invited lectures, awards)
Members of the team were involved in the organisation of one larger research programme in Bonn and two other international conferences (Paris and Toulouse), and gave invited lectures themselves at various other important international conferences.	

Further criterion: 2. Vitality, sustainability and strategy (D2.1-D2.9)

D2.1	Direction in line with the perspective of the planned research directions
The team has described clear plans and goals for future research (in the short term), on black holes, on operadic categories, in representations theory and applications of Galois cohomology, to mention a few.	
D2.2	**Assessment of the previous research objectives and their achievement**
The team has set itself different research goals, e.g. in theoretical physics, theory of operads, information geometry and differential geometry, and has made impressive progress in each of these.	
D2.3	**Assessment of implementation of recommendations from past evaluation**
One of the main recommendations was for the mathematical physics group to strengthen its relation to researchers in the same area at Charles University. This has indeed happened.	
D2.4	**Success in receiving grants**
Besides the Praemium Academiae award to a member of the team, the team has been quite successful in obtaining grants of the CSF.	
D2.5	**Adequacy of instrumental equipment**
For this type of theoretical research, computer equipment and a good library suffice.	
D2.6	**Effectiveness of management**
Not relevant for the team, see the management at Institute level.	
D2.7	**Assessment of professional structure, development strategy and the strategy of keeping best scientists, age structure, career and qualification growth**
The team has succeeded in attracting a considerable number of young scientists at the postdoc level, and might consider coaching member(s) of the team to become future leader(s).	
D2.8	**Creating work-life balance conditions, assessment of approach towards possible gender issues**
These aspects are dealt with at the institute level.	
D2.9	**Relation of the team with regard to the integration, development and sustainability of the research centre funded by the National Programme of Sustainability II.**
Not relevant.	

Further criterion: 3. Cooperation with universities and participation in education (D3.1-D3.6)

D3.1	Scope of cooperation with universities on national and international level
The team gives courses at the university, participates in the Prague Doctoral Programme. Members of the team also gave courses in Padova and Bonn as visiting professors.	
D3.2	**Effectiveness of joint research centres**
The team has participated in the Einstein Centre (with Charles University) already mentioned above.	
D3.3	**Success rate in supervision of PhD students**
(Only) three PhD students defended their theses in the evaluation period.	
D3.4	**Participation of PhD students in the outputs**
The PhD students in the mathematical relativity group made impressive contributions to the publications. In fundamental mathematics, on the other hand, it is unusual for a PhD student to publish in the first few years of his or her PhD studies.	
D3.5	**Participation of the team in master or bachelor studies**
The team lists 11 master courses, no bachelor courses.	
D3.6	**Assessment of cooperation intensity with universities in the form of teaching**
The team should intensity its participation in the teaching of master courses, which should also result in attracting good PhD candidates.	

Further criterion: 4. Outreach activities (D4.1-D4.3)

D4.1	Sufficiency of media strategy and activities in the area of research popularisation
Some members of the team contributed to the popularisation of mathematics and of black holes research.	
D4.2	**Publishing activities and its quality**
The team made various contributions, quite appropriately in the Czech language.	
D4.3	**Participation in professional organisations in the area of research and development**
No activity in this field.	

Recommendations:

- We have been wondering about the cohesion of the Team, and suggest they either tighten the scientific interactions between Physicists and Algebraists, or if this is turns out to be undesirable rethink the Team structure at the Institute level,
- we recommend they increase teaching at the graduate level, and in particular the supervision of master theses, with the aim of attracting more PhD students (see D3.6). The number of PhD students is low and it is important that the members of the

CAS have proper access to master theses supervision and subsequently to PhD students. (We realise that this is partly a matter of the relation between CAS and the University (or universities), but the teams should be more proactive in this respect.)

Other comments of the commission:

3. Differential Equations and Theory of Integral

Introduction:

When assessing the team, its situation after the last evaluation should be taken into account. In this evaluation measures were recommended that either improve the scientific quality in the team, or lead to changes in the team's orientation. The Institute first sought a leader for the team who met its high standards of quality in science and management. High originality and vision were required to reorient the research program in the field of differential-functional equations and to build a new, successful team with a rejuvenated group of talented scientists.

Unfortunately, this attempt did not succeed for various reasons. Therefore, the institute realized an alternative plan to dissolve the team. The research field was traditionally strongly represented at the Institute, with cognate subgroups of well-qualified younger scientists in the teams “Evolution Differential Equations” and “Numerical Analysis”.

The Institute followed the recommendations of the last evaluation in its approach. The process required for this naturally affected the work of the team. The performance of the team had been criticized for the effectiveness of its scientific activities and the level of results obtained. However, it was also considered to be successful in delivering academic education, especially at the Brno branch.

There was an early departure of staff, especially in Brno, where several members were able to continue their work at the university there.

To illustrate the dynamics of the team in the period 2015- 2019 the evolution of its scientific staff is was:

7 members in 2015 + 1 new member in 2019

2 members left in the evaluated period, 2 members decide to leave

The remaining 4 members can be usefully integrated in other teams.

The Commission has attempted to satisfy itself that the Institute's procedures for dissolving the team not only proceeded appropriately under the prevailing conditions, but also led to a positive outcome for those affected. Obviously, some of the questions posed to the Commission in the current assessment are no longer relevant.

Strengths:

The decisions and actions of the Institute made it possible to integrate a relevant research field, that was traditionally strongly represented at the Institute, into a direct cooperation with neighbouring disciplines. Also new impulses could be given e.g. to research in differential-functional equations. This type of systems is increasingly appearing as model equations in relevant applications. and their research opens up very good prospects for talented young scientists who are interested in combining theory and applications.

Weaknesses:

The last evaluation contained as central points of criticism:

- the scientific program is not sufficiently focused on challenging topics in dynamical systems and functional-differential equations
- the connection to real and relevant applications is not strong enough.

While progress has been made on the first point, this has still not been achieved on the second. These criticisms may also have been a reason why the team had no success in recruiting grants so far.

Opportunities:

The team is terminated. Opportunities therefore only arise for those changes that are expected to have positive effects. The initially suboptimal solution B, the finishing of the team and the integration of a remaining part, however, offers also an opportunity for renewal in both the personnel and the research program.

Model equations for real systems very often lead to coupled systems of differential equations and functional equations with deterministic and stochastic terms. The demand for analysis, numerics, simulation and control of such systems, arising in modelling of complex real life systems is increasing. The academy should support research in this direction and urge that it be pursued in a focused manner.

Threats:

The Institute should take care to maintain competence in this research area at an internationally competitive level and strengthen relationships with applications. Missing the opportunities to do so led to a harmful drawback

Main criterion: 1. Quality of results (H1.1-H1.5)

H1.1	Quality of selected outputs of Phase I

The team achieved the following distribution of ratings

1	2	3	1 or 2
11,1%	66,7%	22,2%	77,8%

1 „*world leading*“, 2 „*internationally excellent*“, 3 „*recognized internationally*“

The team improved essentially compared to the last evaluation since 77,8% of the selected outputs were attributed to the top league now compared to 31% before.

H1.2	Contribution of workers on the outputs reached

The number of authors of almost all output is small and the contributions seem to be equally distributed in case of several authors.

H1.3	Quality of all outputs and results

Also here an improvement of the quality can be observed.

H1.4	The most valuable discoveries and findings in the fields, their importance for the field

For many years, research on the Kurzweil-Stieltjes Integral and its applications in very different fields remained a central research in the team and received top recognition. The monograph on this integral, which was completed in the review period, contains many new results on the theory of this integral and on many new applications, justifying in summary to be considered as a compendium of most valuable discoveries in the field and the best contribution of an ending team. The Kurzweil concepts can be used also to generalize the Ito integral for stochastic processes, This Itö-Kurzweil integral can also prove to be a very useful tool in the study of stochastic processes.

H1.5	Contribution of the participation of the authors in large collaborations

There are no large collaborations in the sense of category “D”.

Main criterion: 2. Societal relevance (H2.1-H2.5)

H2.1	Societal relevance of outputs and results pursuant to CAS and institute mission
The outputs cover mainly the basic theory in ordinary differential equations and functional equations, which partially may occur as model equations arising in real life systems. Tools to better understand, control and manage them can be important and of practical use. Therefore, some of the achieved results are relevant also for real life problems. The transfer of mathematical knowledge and methods to solve industrial, economic or social problems played only a minor role in the team - at least so far.	
H2.2	**System functionality for knowledge transfer into practise, its usefulness for society. The impact of the team´s activity on proper practice in society in the area of social sciences and humanities**
Does not apply to the team	
H2.3	**Relation to practice**
The team was not involved in transfer to practice problems	
H2.4	**Participation in AV21 strategy**
The team did not actively participate in AV21	
H2.5	**Cooperation with regions of the Czech Republic**
There is no official cooperation with the regions.	

Further criterion: 1. Position in international and national context (D1.1-D1.3)

D1.1	Comparison of the team with similar international and national institutes
If one restricts oneself to the topics that the team has dealt with in former years, the circle of mathematicians who concentrate on these is comparatively small and forms a closed community that maintains contacts mainly internally. This fact has had a partially negative effect on the comparatively large team at the institute.	
D1.2	**Scope and quality of international and national cooperation and the role of the team in such cooperation; engagement in broad international cooperation**
Within this specialised community, the team and its staff maintained lively scientific contact in all areas and were working directly together in research and education**.**	
D1.3	**Participation of the** workers **in scientific community activities (organizing of conferences and workshops, invited lectures, awards)**
Despite the internal challenges, members of the team actively participated in organizing workshops and conferences focussed on differential-functional-equations.	

Further criterion: 2. Vitality, sustainability and strategy (D2.1-D2.9)

D2.1	Direction in line with the perspective of the planned research directions
The scientific integration of the remaining staff of the previous team into other teams appears to make sense and opens up new perspectives both for the staff and for sub-disciplines in which they will work after the transfer and to which they will contribute their expertise. For example, the research direction Material Memory and Hysteresis in the Evolution Differential Equations Team can expect support from the transfer.	
D2.2	**Assessment of the previous research objectives and their achievement**
In the focus of the research of the team were: Boundary value problems for ordinary differential equations and functional differential equations, asymptotic theory, constructive methods for boundary value problems, integration theory and generalized differential equations. The research treated basic topics including solvability analysis for boundary value problems, qualitative study of solutions, and development of effective tools for construction of solutions, not including development of numerical methods however, but deriving analytic results relevant to the numerical analysis, e.g. for difference equations, including discretization of differential equations. The evaluation of the outputs showed that the results achieved in theory have improved quite well recently. At the same time, however, there was still a deficit in using the potential of the achieved results, e.g. in numerics or in application areas. This also prevented new impulses for the further development of the theory.	
D2.3	**Assessment of implementation of recommendations from past evaluation**
The Institute followed consequently the recommendation of the last evaluation and performed all the necessary. The steps made and leading to closing the team, not as an aim but as final result, were necessary. The team did not get a leader. It was successful in improving the quality of the outputs, but could not succeed in removing further deficits. The continuing fact that the team is staying without grants is a crucial indicator that a change was necessary.	
D2.4	**Success in receiving grants**
The team itself has recognised that a particular weakness has remained: the lack of grants. Despite an understanding of the team's precarious position, it is hard to see why it did not take the serious criticism in the last evaluation seriously enough: *"Transfer of mathematical knowledge and methods to solve industrial, economic or social problems does practically not exist. The fact that the team had no substantially funded project, neither in theoretical, nor in applied research, has to be seen as very critical*."	
D2.5	**Adequacy of instrumental equipment**
Does not apply to the team.	
D2.6	**Effectiveness of management**
Does not apply to the team.	
D2.7	**Assessment of professional structure, development strategy and the strategy of keeping best scientists, age structure, career and qualification growth**

As mentioned in the introduction the Institute tried to find a strong leader and to change the age and quality structure of the team, but was not successful in hiring a top scientist as leader, a necessary condition to continue the team.	
D2.8	**Creating work-life balance conditions, assessment of approach towards possible gender issues**
Does not apply to the team.	
D2.9	**Relation of the team with regard to the integration, development and sustainability of the research centre funded by the National Programme of Sustainability II.**
Not relevant.	

Further criterion: 3. Cooperation with universities and participation in education (D3.1-D3.6)

D3.1	**Scope of cooperation with universities on national and international level**
Members of the team participated in teaching bachelor, master and PhD students at several universities through lectures and supervision of students.	
D3.2	**Effectiveness of joint research centres**
Does not apply to the team.	
D3.3	**Success rate in supervision of PhD students**
In the team, 8 PhD students were supervised in the reviewed period, 5 students successfully completed their doctorates. Considering the size of the team, this seems to be a comparatively good result for an institute of the Academy.	
D3.4	**Participation of PhD students in the outputs**
To comment on this point needs more information.	
D3.5	**Participation of the team in master or bachelor studies**
The team participated comparatively well in basic lectures in the Bachelor's program. Early contact with students is a decisive advantage in competition for talented students.	
D3.6	**Assessment of cooperation intensity with universities in the form of teaching**
The cooperation with universities in teaching deserves acknowledgement. The rather small number of lectures for Master's students remains incomprehensible.	

Further criterion: 4. Outreach activities (D4.1-D4.3)

D4.1	**Sufficiency of media strategy and activities in the area of research popularisation**
Does not apply to the team	
D4.2	**Publishing activities and its quality**

Does not apply to the team	
D4.3	**Participation in professional organisations in the area of research and development**
Does not apply to the team	

Other comments of the commission:

We note that the Differential Equations and Theory of Integral (DETI) Team has been dissolved and its members are being redistributed to other Teams. We therefore make a null return under the recommendation of change of level of support. However, we do recommend that resource adjustments made to the Teams into which the former members of DETI are transferred.

The Commission supports the Institute's approach, which, under the present circumstances, has found a solution that offers good prospects both for the subject area and for the staff remaining at the Institute. The visible success in the assessment in Phase I should partly compensate for the burden of integrating what is worth preserving into other teams.

4. Evolution Differential Equations

Strengths:

The scientific productivity and international visibility and recognition of the team is very high which is documented by the evaluation of Phase I results. The team is active in receiving grants (ERC-grant by E. Feireisl, Neuron Impuls grant by O. Kreml and several grants from CSF). The team members are developing new research directions (e.g., stochastically forced compressible fluid flows). Moreover, the team has rich teaching activities at Czech universities and abroad and summer schools. PhD students are involved in publications in top journals. The team is highly visible in the scientific community and active through several memberships in scientific boards and councils. Although the focus is on basic research there is a cooperation with an industrial partner. The team is active in the Necas Center for mathematical modelling.

Weaknesses:

Despite its high scientific reputation, the team has problems in hiring talented young people after their PhD. According to the team's report the main reasons are the unclear way of planning a scientific career in combination with very low salaries.

Opportunities:

The team consists of several talented young researchers visible in both areas, fluids and solids which exhibits a potential for new team leaders. The cooperation with industrial partners is very promising which is not usual for a basic research-oriented team.

Threats:

The key persons are close to the retirement age and although there are several younger recognized researchers, there is an age gap in the team structure. The success of attracting talented young people (PhD students, postdocs and tenure track positions) exhibits the main threat of the team.

Main criterion: 1. Quality of results (H1.1-H1.5)

H1.1	Quality of selected outputs of Phase I
The Department Evolution Differential Equations was highly successful in the rating of the 26 outputs it selected, as the following distribution of ratings shows 1 2 3 1 or 2 30,8% 57,7% 11,5% 88,5% 1 „*world leading*“, 2 „*internationally excellent*“, 3 *„recognized internationally*“ With this it is above the institute's average. Approximately half of the number of papers was written in collaborations with international co-authors. All publications appeared in highly recognized international journals mainly in the field of applied mathematics. (e.g. Archive for Rational Mechanics and Analysis, SIAM Journal Mathematical Analysis, Journal de Mathematiques Pures et Appliques, Nonlinear Analysis: Theory, Methods and Applications)	
H1.2	**Contribution of workers on the outputs reached**

The chosen outputs for Phase I show a clear focus on topics related to fluid mechanics and hence members of the team working in this field dominate the list of authors. The by far smaller subgroup working on solid mechanics also contributed with strong papers to the output submitted for Phase I.

H1.3	Quality of all outputs and results

The group is extremely vital and productive – in particular in the field of fluid dynamics, where it covers a wide range of topics from the modelling, the analysis (existence, regularity, singular limits, stability, stochastic terms, decay properties) up to the development of appropriate numerical methods. In many cases (e.g. for the treatment of stochastic effects) new original theories were developed that are valuable also in other contexts. Several monographs have been published on fluid related topics.

In the context of solid mechanics the group continued its strong work on contact problems and on phenomena including hysteresis. Here, models for temperature dependent processes and for processes in porous media played a central role. The power of the Kurzweil integral for modelling discontinuous processes was highlighted in several papers. This observation is highly relevant for the entire community modelling rate-independent processes with discontinuities in time.

H1.4	The most valuable discoveries and findings in the fields, their importance for the field

The most valuable discoveries comprise the incorporation of stochastic effects in fluid models and their thorough analysis. For that purpose a completely new theory was developed and published – among others – in the monograph Breit/Feireisl/Hofmanova, Stochastically forced compressible fluid flows, De Gruyter, 2018.

Further important contributions concern the proof of non-uniqueness properties of solutions to certain fluid models, existence theorems for fluid structure interaction problems and the development of a model for fluid flow and phase transition in porous media. All results open new research directions that are relevant for a broader community.

H1.5	Contribution of the participation of the authors in large collaborations

There are no large collaborations in the sense of category “D”.

Main criterion: 2. Societal relevance (H2.1-H2.5)

H2.1	Societal relevance of outputs and results pursuant to CAS and institute mission

The main focus of the group is on basic research in mathematics with special focus on the modelling of fluids, fluid structure interactions and the modelling of multi-processes in solids. Only with such research as a basis it is possible to develop numerical models and methods that are mathematically sound (e.g. have guaranteed convergence properties to the correct solutions) and that allow for realistic and reliable simulations for the prediction of the properties/behaviour of concrete engineering structures and devices. The group develops exactly this basis and at the same time is actively involved in the transfer of the theoretical findings to – at least prototypical – numerical methods.

A particular highlight is the cooperation with the Doosan-Bobcat company where in a continued mutual exchange real world problems motivate research questions in the group and where the scientific background of the group helps to improve the models used in the Doosan-Bobcat company.

H2.2	System functionality for knowledge transfer into practise, its usefulness for society. The impact of the team´s activity on proper practice in society in the area of social sciences and humanities
The results of the group benefit mainly indirectly social aspects of society. As mentioned in the previous section (H2.1) the main topic of the group is the mathematical foundation of fluid flow, fluid structure interactions and of complex materials. Thereby improving the tools available to others who do develop innovations that benefit society.	
H2.3	**Relation to practice**
The group is active in the mathematical modelling and analysis of fluids and complex processes in solids. This basic research is relevant for a multitude of applications like for instance for models in medicine and biology (e.g. blood flow through vessels) or the design of new multi-functional materials (e.g. ferroelectric materials with special properties). In this sense the group has high potential to participate directly in the solution of practical problems. Through the cooperation with the Doosan-Bobcat company the group proves its ability for such cooperations.	
H2.4	**Participation in AV21 strategy**
Institute of Mathematics participates in the research program “Hopes and risks of the digital era” of AV21 strategy. Team members have organised regular workshops Mathematics for Industry (September 2017, October 2018, November 2019) in cooperation with engineers from the Doosan-Bobcat company. The team organised two workshops "Mathematics for industry" in 2018 and 2019 by EU-MATHS-IN.cz network which were supported by the programme AV21.	
H2.5	**Cooperation with regions of the Czech Republic**
The group does not have official cooperation agreements with other regions. However, it is highly active and connected with other regions on the basis of the individual contacts of its members.	

Further criterion: 1. Position in international and national context (D1.1-D1.3)

D1.1	Comparison of the team with similar international and national institutes
The team is very good in comparison with similar international institutes. Particularly, E. Feireisl is a world recognized key person in the area of mathematical modelling and theory of partial differential equations. Moreover, the team consists of several others recognized experts with an excellent publication record.	
D1.2	**Scope and quality of international and national cooperation and the role of the team in such cooperation; engagement in broad international cooperation**
There is a lot of intensive and fruitful international cooperations of the team members with researchers all over the world. The international cooperation is based mostly on mutual connections among particular partners. The team participates in the activities of the national Nečas Center for Mathematical Modelling. The team members are active in several scientific councils, committees and editorial boards.	
D1.3	**Participation of the workers in scientific community activities (organizing of conferences and workshops, invited lectures, awards)**

The team members are active in organizing conferences, workshops and summer schools, some of them have a very recognized reputation, e.g., SIAM Conference of Analysis of PDEs, EMS Summer School in Applied Mathematics.

Further criterion: 2. Vitality, sustainability and strategy (D2.1-D2.9)

D2.1	Direction in line with the perspective of the planned research directions
The planned research directions are in line with the research in the evaluated period, namely analysis of equations describing compressible and incompressible fluids, fluid-structure interaction, solid mechanics and bifurcation.	
D2.2	**Assessment of the previous research objectives and their achievement**
The activity plan for the evaluated period was successfully fulfilled and some new research directions were opened (e.g., stochastic aspects of continuum mechanics, multiscale problems). The achieved results were published in recognized journals, often with foreign co-authors.	
D2.3	**Assessment of implementation of recommendations from past evaluation**
The department was active in the implementation of all recommendations from the past evaluation. An activity in all listed items is documented, the most of recommendations were successfully implemented. However, the activities in the Nečas center and the cooperation in the field of numerical mathematics can be still strengthened.	
D2.4	**Success in receiving grants**
The department is successful in receiving grants, the main success is the ERC Advanced Grant obtained by E. Feireisl in 2012-2018. Moreover, the team members obtained several grants of the Czech Science Foundation and also Neuron Impuls grant by O. Kreml.	
D2.5	**Adequacy of instrumental equipment**
The Institute of Mathematics provides a well-equipped library including the access to e-journals and databases, which is sufficient since the majority of the research belongs to theoretical science.	
D2.6	**Effectiveness of management**
The management organization of the department is on a good level, the research of particular team members and their pedagogical and outreach activities are well balanced. Some senior researchers are approaching retirement age so it is a challenge for future years to keep the level of the team.	
D2.7	**Assessment of professional structure, development strategy and the strategy of keeping best scientists, age structure, career and qualification growth**
The structure of the team is reasonably balanced, several younger persons form an important part of the department. Due to the grants, several postdocs were hired for several years. Moreover, by cooperating with universities, the team attracts a few PhD students. The team also succeeded to obtain financial support for two postdocs from the Perspective Human Resource Programme of the Czech Academy of Sciences.	
D2.8	**Creating work-life balance conditions, assessment of approach towards possible gender issues**

One of the weaknesses written in the self-evaluation report concerns the difficulties with combining the private life and scientific career. Young and talented researchers have to move every two years from one postdoc place to another before getting a tenure. Group members with family care duties are supported according to their individual needs.

D2.9	Relation of the team with regard to the integration, development and sustainability of the research centre funded by the National Programme of Sustainability II.
Not relevant.	

Further criterion: 3. Cooperation with universities and participation in education (D3.1-D3.6)

D3.1	Scope of cooperation with universities on national and international level
The cooperation with universities on national as well as international level is on a high level. The team members teach regularly at the two Charles University and Czech Technical University in Prague and West Bohemian University in Pilsen. Moreover, several lectures were given occasionally in University of Pittsburgh, Botswana International University of Science and Technology and Universität Würzburg. Further, many short courses were given at several universities.	
D3.2	**Effectiveness of joint research centres**
Team is involved in Nečas Center for Mathematical Modelling (see item D1.2 and D1.3). Although the Nečas Center was active in the evaluation period, its activities could be strengthened.	
D3.3	**Success rate in supervision of PhD students**
Team is active in the supervising of PhD students in cooperation with the national and foreign universities. Some of the students come from outside of Czech Republic.	
D3.4	**Participation of PhD students in the outputs**
PhD students are fully involved in the outputs of the departments, some articles co-authored by PhD students belong to the top journals.	
D3.5	**Participation of the team in master or bachelor studies**
Team members give a lecture at master and bachelor studies and occasionally they supervise master and bachelor thesis.	
D3.6	**Assessment of cooperation intensity with universities in the form of teaching**
The intensity of the cooperation with universities in the form of teaching is on the reasonable level. There are not enough students (and therefore teaching opportunities) motivated by studying mathematics.	

Further criterion: 4. Outreach activities (D4.1-D4.3)

D4.1	Sufficiency of media strategy and activities in the area of research popularisation
The activities in the area of research popularisation are on a sufficient level, e.g., the popularization of the science by the Fields medal winner Cedric Villani in Czech Television and several other events.	
D4.2	**Publishing activities and its quality**
No special publishing outreach activity was documented in the report and the presentation during the on-line meeting.	
D4.3	**Participation in professional organisations in the area of research and development**
No activity in this item is documented.	

Recommendations:

1. Continue in the cooperation with the teams and researchers dealing with numerical mathematics and stochastics.
2. Keep and possibly extend the cooperation with other institutes and universities, particularly within Nečas Center for Mathematical Modelling.
3. Keep and possibly extend the cooperation with industrial partners.
4. Continue in the effort of attracting young researchers since several key persons are close to the retirement age.

Other comments of the commission:

5. Mathematical Logic and Theoretical Computer Science

Strengths:

The group has a very strong research output, with many papers in high quality journals. The research covers a broad research spectrum, ranging from set theory (foundations of mathematics) to questions of complexity of computation. They have been successful in obtaining major grants, including an ERC advanced grant (obtained in the previous evaluation period, but active in the current one).

Weaknesses:

Small number of PhD students. Lack of long-term strategy.

Opportunities:

Some subgroups might profit from more interaction with other teams, both in Mathematics (notably Abstract Analysis) and in Computer Science.

Threats:

Publications of the team are often in pure mathematics journals or computer science proceedings, for which bibliometric data such as impact factors and citations are misleading. The age distribution seems to be a bit vulnerable. The team leader is among the strongest and internationally most well-known members but will retire within the coming evaluation period.

Main criterion: 1. Quality of results (H1.1-H1.5)

H1.1	Quality of selected outputs of Phase I
The team achieved the following distribution of ratings	
1	2	3	1 or 2
52,0%	44,0%	4,0%	96,0%
1 „*world leading*“, 2 „*internationally excellent*“, 3 „*recognized internationally*“	
Excellent output, as witnessed by the tables we have been provided. (96% in 1+2), publications in leading journals.	
H1.2	**Contribution of workers on the outputs reached**
Members of the team authored or co-authored 84 articles in international journals, and 42 articles in Proceedings of conferences. (See table.) All papers have major contributions by team members.	
H1.3	**Quality of all outputs and results**
Most of the articles mentioned above are world-class contributions. It should be stressed that the team fully justifies its title, since a fair number of these contributions relate to mathematical logic and fundamental mathematics and to theoretical computer science as well. Cross-collaboration with the Institute of Computer Science of the CAS is effective and fruitful.	
H1.4	**The most valuable discoveries and findings in the fields, their importance for the field**

Major advances in complexity theory, e.g. results on exponential lower bounds of proofs (Hrubes and Pudlak). In set theory, advances on the existence of and structure of (the set of) P-points. A series of papers by Hladky and collaborators (among whom Piguet from CAS-Comp Sci) established a slight weakening of a conjecture of Erdos on graphs containing all trees of a certain type.

As a good example of collaboration across institutes, we mention the important article "Packing degenerate graphs" (Advances in mathematics 2019, 57 pages) which is a remarkable piece of mathematics, and an example to follow. This work is an international collaboration in more than one way: two of the authors (Peter Allen, Julia Böttcher) are from the London School of Economics, Jan Hladky is a researcher at the Mathematics Institute of the CAS and Diana Piguet, originally from Switzerland, is a researcher at the Computer Science Institute of the CAS. Moreover, the article contributes to graph theory at the highest level, by providing positive results on packing of graphs with possibly high maximum degrees, including near-perfect packing results. It is therefore an important work on fundamental but readily applicable mathematics.

H1.5	Contribution of the participation of the authors in large collaborations
There are no large collaborations in the sense of category “D”.	

Main criterion: 2. Societal relevance (H2.1-H2.5)

H2.1	Societal relevance of outputs and results pursuant to CAS and institute mission
Research is mainly fundamental, but the topics are central and relate to other parts of mathematics. Societal relevance is mainly through connections to computer science.	
H2.2	**System functionality for knowledge transfer into practise, its usefulness for society. The impact of the team´s activity on proper practice in society in the area of social sciences and humanities**
Fundamental mathematics often has only indirect technical impact on social sciences and humanities. However, the rigorous procedures mathematicians use for showing their results deserve to be better known at a time when asserting without proof is fashionable. In this respect mathematicians may have an immediate impact on society. For teams working (also) on graph theory, like this one, it should be mentioned that graphs occur in all uses of networks and network analysis, and have ubiquitous societal relevance.	
H2.3	**Relation to practice**
Mathematics and computer science are used in every industry, including banks and the stock exchange but no special specific information has been provided by the team.	
H2.4	**Participation in AV21 strategy**
No specific information has been provided by the team.	
H2.5	**Cooperation with regions of the Czech Republic**
There is no official cooperation with the regions.	

Further criterion: 1. Position in international and national context (D1.1-D1.3)

D1.1	Comparison of the team with similar international and national institutes
Many members of the team operate at the highest international level in their discipline, see also the next question.	
D1.2	**Scope and quality of international and national cooperation and the role of the team in such cooperation; engagement in broad international cooperation**
The team has many ongoing international collaborations with members of established research institutes and universities, e.g. in the UK, Germany, Poland, Austria.	
D1.3	**Participation of the workers in scientific community activities (organizing of conferences and workshops, invited lectures, awards)**
Members of the team are active as editors or associate editors of international journals. The team leader served on the program committee of the 8th European Congress of Mathematics.	

Further criterion: 2. Vitality, sustainability and strategy (D2.1-D2.9)

D2.1	Direction in line with the perspective of the planned research directions
Progress in fundamental science is often difficult to plan ahead, both in direction and in schedule. Nonetheless, the team set itself explicit targets in various directions and has made good progress in each of these, see D2.2	
D2.2	**Assessment of the previous research objectives and their achievement**
Considerable progress as planned was made, among others, on NP search problems, descriptive set theory, set theoretic forcing, communication complexity, control of discrete event systems, and expander graphs.	
D2.3	**Assessment of implementation of recommendations from past evaluation**
The team was advised to maintain the high quality of its research, intensify collaboration with colleagues working in complexity theory at Charles University, and possibly to increase international collaboration. They surely succeeded to follow the first recommendation. The team has also been successful in the other two directions, obtaining a joint EXPRO grant with Charles University, and having been able to attract internationally renowned researchers as E. Cech Visiting Professors, Rödl in graph theory and Todorcevic in set theory.	
D2.4	**Success in receiving grants**
During the evaluated period 2015-2019, several new grants were obtained by team members: An EXPRO Grant from the Czech Science Foundation (P. Hrubes), two international Grants (D. Chodousky with Austria, J. Komenda with Germany), three standard grants from the CSF or the Ministry of Education (J. Hladky, E. Jerabek, J. Komenda). In addition, the ERC grant of Pudlak was still running for part of the evaluation period.	
D2.5	**Adequacy of instrumental equipment**
Not applicable.	

D2.6	Effectiveness of management
Management seems to mainly take place at the institute level, and the structure of the team itself is relatively loose but effective.	
D2.7	**Assessment of professional structure, development strategy and the strategy of keeping best scientists, age structure, career and qualification growth**
The team is well structured, covering different but related areas. However, the age distribution could be better, and they should work on future leadership, see the SWOT analysis above.	
D2.8	**Creating work-life balance conditions, assessment of approach towards possible gender issues**
Seems to be dealt with adequately at institute level.	
D2.9	**Relation of the team with regard to the integration, development and sustainability of the research centre funded by the National Programme of Sustainability II.**
Not relevant.	

Further criterion: 3. Cooperation with universities and participation in education (D3.1-D3.6)

D3.1	Scope of cooperation with universities on national and international level
Cooperation in research is mostly at an individual level, but is widespread, as mentioned before.	
D3.2	**Effectiveness of joint research centres**
Not relevant.	
D3.3	**Success rate in supervision of PhD students**
In the period under evaluation, the team supervised or is supervising 8 students, 2 of whom graduated in this period.	
D3.4	**Participation of PhD students in the outputs**
The PhD students actively contributed to the research output, as authors or co-authors.	
D3.5	**Participation of the team in master or bachelor studies**
Members of the team gave a considerable number of lectures (37 semester courses) at Charles university, and supervised a master thesis and a bachelor thesis.	
D3.6	**Assessment of cooperation intensity with universities in the form of teaching**
The cooperation at the university level is good, and the number of courses taught seems appropriate. It would be good to attempt to enlarge the number of master thesis supervisions, with the aim of attracting more PhD students.	

Further criterion: 4. Outreach activities (D4.1-D4.3)

D4.1	Sufficiency of media strategy and activities in the area of research popularisation
The team participated in various important activities in the direction of diffusion of mathematics, released a now widely used speech synthesizer, and developed a game that can be played on smartphones.	
D4.2	**Publishing activities and its quality**
The game and synthesizer mentioned above are a form of publishing, of high quality.	
D4.3	**Participation in professional organisations in the area of research and development**
Not relevant.	

Recommendations:

- mirroring the recommendation for Abstract Analysis team above, to increase mutual cooperation,
- there are also good opportunities for intensifying interaction with Theoretical Computer Science (of the Computer Science Institute),
- to prepare a detailed and ambitious hiring strategy for replacing Pavel Pudlak when he retires,
- we recommend they increase teaching at the graduate level, and in particular the supervision of master theses, with the aim of attracting more PhD students (see D3.6). The number of PhD students is low and it is important that the members of the CAS have proper access to master theses supervision and subsequently to PhD students. (We realise that this is partly a matter of the relation between CAS and the University (or universities), but the teams should be more proactive in this respect.)

Other comments of the commission:

6. Numerical Analysis

Strengths:

Numerical analysis is a highly important subject within mathematics and it interfaces with computer science. It provides a profound basis for the development of reliable simulation tools that in turn play an enabling technology for many applications in science and in industry. The Numerical Analysis group conducts research in fundamental numerical analysis, in particular in numerical linear algebra. This is in combination with the development of models and discretization schemes in several application areas. Another focus area is the development of domain decomposition schemes for parallel computing.

- group grew from 3.4 FTE to 6 FTE + 1 PhD student
- during the evaluation period the group enriched and strengthened its research topics in the field of numerical linear algebra and numerical methods for Navier-Stokes equations (hence enabling a close collaboration with the Evolution Differential Equation group).
- The group conducts deep numerical analysis in combination with the development of prototypic codes in several application areas. This combination should be further followed.
- Good age structure with several young researchers who could take over scientific leading positions
- long-term visits of young group members abroad in order to broaden their scientific knowledge
- participation at MATH-IN-HPC.EDU (doctoral school on HPC with TU Ostrava, Charles University)
- despite its smallness the group is successful with several project proposals (e.g. Neuron Impuls project, T. Vejchodsky, several grants from CSF, but mainly concentrated on two members of the team)

Weaknesses:

While strong in traditional fields of numerical analysis, several new directions remain unexplored, e.g. uncertainty quantification, multilevel methods, reduced order modelling, and links to machine learning and artificial intelligence research. In particular the link to high performance computing is not fully exploited.

Opportunities:

- The interdisciplinary potential of the group (physics, engineering, scientific computing, biology, chemistry) could be further explored. The group collaborates with applied partners mainly in academia. Strengthening the collaboration with application fields in science or engineering or collaboration with an industrial partner could enrich the research.
  - The team is well integrated in the Necas Center. However, activities could be intensified (joint workshops, seminars etc. also with other institutes)
  - A closer collaboration IT4I in Ostrava could strengthen the efforts in parallel computing. This would be important to increase the impact of research of the department for CAS.
  - There is high potential in performing joint research with the EDE team on fluid dynamics.

Threats:

- problems in finding PhD students (structural problem)
- two of the group members serve the whole institute as director and as deputy director (= less time for research)

Main criterion: 1. Quality of results (H1.1-H1.5)

H1.1	Quality of selected outputs of Phase I
The group Numerical Analysis was successful in the rating of the 10 outputs it selected, as the following distribution of ratings shows 1 2 3 1 or 2 10% 70% 20% 80% 1 „*world leading*“, 2 „*internationally excellent*“, 3 „*recognized internationally*“ Most papers were written in collaborations with international co-authors. The publications appeared in highly recognized international journals. (e.g. SIAM Journal on Scientific Computing, Journal of Computational Physics, Mathematical Models and Methods in Applied Sciences)	
H1.2	**Contribution of workers on the outputs reached**
All members of the group contributed to the selected outputs. With approximately 70 articles in journals and four books the small group (7 researchers) is very productive. This is even more remarkable since two members of the group serve as director and as deputy director of the whole institute.	
H1.3	**Quality of all outputs and results**
The aim of the group is to perform excellent internationally recognized research in Numerical Analysis. In view of the scientific outputs the group is on a very good path towards this aim. In the evaluation period the team strengthened and broadened its competences in several fields (e.g. numerical linear algebra, computational fluid mechanics) that are relevant for the success of the whole team.	
H1.4	**The most valuable discoveries and findings in the fields, their importance for the field**
The group acts on an internationally competitive level and obtained a variety of valuable results. However, it is difficult to single out some outstanding achievement. We mention here the strong contributions in numerical linear algebra and specifically algorithms for saddle point systems, as well as the work on parallel domain decomposition methods.	
H1.5	**Contribution of the participation of the authors in large collaborations**
The authors do not participate in large collaborations in the sense of category „D“.	

Main criterion: 2. Societal relevance (H2.1-H2.5)

H2.1	Societal relevance of outputs and results pursuant to CAS and institute mission
Nearly all technical innovations rely on computational simulations of the relevant underlying processes. The research of the group is basis oriented and provides the mathematical	

foundation for such simulations. It develops discretization strategies for different applications in physics, engineering, scientific computing, biology and chemistry.

The group performs fundamental research in numerical mathematics and its applications and fits perfectly to the mission of the Institute of Mathematics and the CAS.

H2.2	**System functionality for knowledge transfer into practise, its usefulness for society. The impact of the team´s activity on proper practice in society in the area of social sciences and humanities**

The group performs basic research in numerical analysis that – as described above – is at the core for the development of a multitude of further innovations in engineering, chemistry and biochemistry. Hence, there is a sort of indirect contribution of the group to the area of social sciences and humanities. However, we point out that the impact outside mathematics depends fundamentally on the realization of the methods in sustainable (and often parallel) software. If not addressed within the department itself, this should be sought in collaboration e.g. with other institutes of the CAS (Computational Mathematics) or e.g. IT4I in Ostrava.

H2.3	**Relation to practice**

As mentioned before, the group performs basic research in numerical analysis. Several research questions are motivated by and related to applications that come from fluid mechanics, mathematical biology, physics and chemistry. Currently the group transfers its knowledge mainly to non-mathematical partners in academia. A collaboration with one industrial partner on one of the above topics could enrich the research focus of the group.

H2.4	**Participation in AV21 strategy**

The Institute of Mathematics participates in the research program “Hopes and risks of the digital era” of AV21 strategy. A possible further topic for participation could be „Efficient Energy Conversion and Storage“.

H2.5	**Cooperation with regions of the Czech Republic**

The group is a partner in the doctoral school in high performance computing MATH-IN-HPC.EDU (TU Ostrava, Charles University), a cooperation that is absolutely important for attracting PhD students and young academics to the group. A collaboration with IT4I is present but it appears that this could be expanded for mutual benefit.

Further criterion: 1. Position in international and national context (D1.1-D1.3)

D1.1	**Comparison of the team with similar international and national institutes**

The department is conducting research on an international level.

D1.2	**Scope and quality of international and national cooperation and the role of the team in such cooperation; engagement in broad international cooperation**

The researchers are well connected to the international research community in their fields.

D1.3	**Participation of the workers in scientific community activities (organizing of conferences and workshops, invited lectures, awards)**

The department members participate regularly in workshops or conferences, they are also active in organizing them.

Further criterion: 2. Vitality, sustainability and strategy (D2.1-D2.9)

D2.1	Direction in line with the perspective of the planned research directions
The research directions can be classified as central topics in the field of numerical mathematics.	
D2.2	**Assessment of the previous research objectives and their achievement**
The department has been very successful in their previous research activities.	
D2.3	**Assessment of implementation of recommendations from past evaluation**
The previous recommendations have been followed within the given constraints.	
D2.4	**Success in receiving grants**
The department has been successful in receiving national grants. An increased activity e.g. on the EU level would strengthen the department further.	
D2.5	**Adequacy of instrumental equipment**
For most activities the equipment is adequate. However, we repeat here the structural deficit in high performance computing. While not immediately needed for most mathematical research per se, the larger impact of this research is impeded by the underdeveloped link to high performance computing.	
D2.6	**Effectiveness of management**
The department appears well managed.	
D2.7	**Assessment of professional structure, development strategy and the strategy of keeping best scientists, age structure, career and qualification growth**
The team encourages young researchers/young team members to spend a certain time abroad (and come back to the Institute) in order to broaden their scientific fields.	
D2.8	**Creating work-life balance conditions, assessment of approach towards possible gender issues**
This is a topic for the institute level.	
D2.9	**Relation of the team with regard to the integration, development and sustainability of the research centre funded by the National Programme of Sustainability II.**
Not relevant.	

Further criterion: 3. Cooperation with universities and participation in education (D3.1-D3.6)

D3.1	Scope of cooperation with universities on national and international level
These activities are on the expected level.	
D3.2	**Effectiveness of joint research centres**
The interface with the Necas center could possibly be strengthened further.	

D3.3	Success rate in supervision of PhD students
During the evaluation period the team supervised two PhD students.	
D3.4	**Participation of PhD students in the outputs**
PhD students regularly appear as co-authors on journal papers and in proceedings.	
D3.5	**Participation of the team in master or bachelor studies**
The team regularly teaches bachelor and master courses at Universities. In the evaluation period, two bachelor thesis and two master thesis were finished under the supervision of team members.	
D3.6	**Assessment of cooperation intensity with universities in the form of teaching**
See D3.5.	

Further criterion: 4. Outreach activities (D4.1-D4.3)

D4.1	Sufficiency of media strategy and activities in the area of research popularisation
The team is very active in popularising research: two monographs, several articles and interviews addressing a broader audience were published. The team regularly contributes to events like the Day of Open Doors.	
D4.2	**Publishing activities and its quality**
See D4.1.	
D4.3	**Participation in professional organisations in the area of research and development**
Team members regularly organize conferences and summer schools, they are active in editorial boards of mathematical journals and serve as experts in scientific commissions.	

Other comments of the commission:

We note that CAS has lost the important field of Numerical Linear Algebra, since the strong team, led by Z. Strakoš has left CAS to join the Department of Mathematics at Charles University. If CAS considers restarting this activity, then the Numerical Analysis Team would form a good nucleus for it.

Final report was elaborated by:

Commission 1 - Mathematics and computer sciences
Evaluated teams No.: 1, 2, 3, 4, 5, 6

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