

Teze disertace
k získání vědeckého titulu “doktor věd”
ve skupině věd “Fyzikálně - matematické vědy”

**Chirally motivated Prague model
for $\bar{K}N$ and ηN interactions
and its applications**

Komise pro obhajoby doktorských disertací v oboru:
“Jaderná, subjaderná a matematická fyzika”

Jméno uchazeče: Aleš Cieplý

Pracoviště uchazeče: Ústav jaderné fyziky AV ČR, v. v. i.

Místo a datum: Řež, červenec 2024

Preface

The thesis recaps the author's contributions to the development of the so called Prague model designed to describe the low-energy interactions of pseudoscalar mesons with the SU(3) octet baryons. The listed papers deal with $\bar{K}N$ and ηN interactions, with strong couplings to other meson-baryon channels derived from chiral Lagrangian, and demonstrate the Prague model suitability in its applications to meson-nuclear interactions and to meson-baryon rescattering in the final state of photoproduction reactions.

Keywords: chiral symmetry, meson-baryon interactions, meson-nuclear interactions, coupled channels, baryon resonances

Contents

1	Introduction	1
2	Methodology	3
3	Author's contributions to the topic	6
3.1	$\bar{K}N$ interactions	6
3.2	ηN and $\eta' N$ interactions	11
3.3	$K^- N$ and ηN systems in nuclear matter	12
3.4	$\pi\Sigma$ photoproduction	18
4	Bibliography	21

1 Introduction

The modern treatment of meson-baryon (MB) interactions at low energies is based on Chiral Perturbation Theory (ChPT), an effective field theory that implements the QCD symmetries in the region of the large strong coupling constant [1–3]. The power counting issues arising from the large nucleon mass in the chiral limit can be overcome by various methods and the same applies to problems with convergence of the perturbation series due to appearance of resonances in the studied energy region. Such an approach combined with multi-channel techniques was used in Ref. [4] for the first time in the sector involving strange hadrons. There, effective pseudo-potentials were constructed to match the chiral amplitudes in the Born approximation and Lippmann-Schwinger equation then allowed to sum properly a presumably dominant part of the full perturbation series. Another way of dealing with the divergences was adopted in [5] where the intermediate state Green function was regularized by means of a momentum cutoff. Finally, the chiral approach to $\bar{K}N$ interactions was reformulated within a complete quantum field methodology based on dispersion relation for the inverse of the scattering matrix and dimensional regularization was used to tame the infinities of the MB loop function [6, 7]. The approach is often referred to as the chiral unitary model as it preserves unitarity at each order of the chiral expansion of the MB potential.

A broad energy region around the $\bar{K}N$ threshold is strongly influenced by the $\Lambda(1405)$ resonance. While its properties have been well known for a long time, the nature of the resonance retains some mysteries. The chiral approaches that implement the strong $\bar{K}N-\pi\Sigma$ coupling generate the $\Lambda(1405)$ dynamically and predict that it is composed of two close resonant states, each of them coupling individually to the $\bar{K}N$ and $\pi\Sigma$ channels [6, 7]. More on this so-called double-pole structure of $\Lambda(1405)$ and related matters can be found in reviews [8, 9].

In parallel to the strangeness $S = -1$ sector, similar models were developed for the $S = 0$ sector to describe the ηN interactions as well [10, 11]. There, the $N(1535)$ and $N(1650)$ resonances are generated dynamically within a framework of two-body MB coupled channels interactions derived from a chiral Lagrangian.

A realistic description of the MB systems at low energies is a prerequisite for their proper treatment in nuclear medium, e.g. when describing meson-nuclear reactions, or for dealing with processes that involve MB rescattering in the final state. In nuclear matter, the meson-nuclear optical potential is often constructed from the in-medium MB amplitudes, so the density dependence of the latter plays a major role in addition to the energy dependence. Such K^- -nuclear potentials were successfully used in calculations of the energy shifts and widths due to strong interaction in kaonic atoms [12]. An attractive character of the $\bar{K}N$ and ηN interactions at energies below the

respective MB thresholds has also motivated theoretical predictions and searches for K^- - and η -nuclear quasi-bound states [13, 14]. Another natural application is also represented by studies of the equation of state of neutron stars in relation to the strangeness, see the comprehensive review [15] on many aspects related to strangeness in nuclear physics.

2 Methodology

The Prague approach (this name has become common in the literature after its first appearance in [ACi13]) to $\bar{K}N$ and ηN coupled channels interactions has been developed over the years building on the original separable potential models presented in [4, 10]. Its effective potentials $v_{c'c} = f_{c'c}^{\text{tree}}$ are constructed to match the tree level MB amplitudes derived up to the NLO of the chiral Lagrangian and we use the channel notation $c = jb$ (meson j , baryon b). These potentials are extended off-the-energy shell by means of the Yamaguchi form factors $g_c(k)$ featuring regulator scales (*soft cutoffs*) α_c . The unitarization and summation of the main part of the perturbative series is then achieved by solving a multichannel Lippmann-Schwinger integral equation for the scattering amplitude

$$f_{c'c}(W) = g_{c'}(k') [(1 - v \cdot G(W))^{-1} \cdot v]_{c'c} g_c(k), \quad (1)$$

where the intermediate state MB loop function can be written as

$$G_c(W) = -4\pi \int \frac{d^3 p}{(2\pi)^3} \frac{g_c^2(p^2)}{k_c^2 - p^2 + i0} \quad (2)$$

and we introduced the energy $W = \sqrt{s}$ and on-shell momenta k_c in the MB c.m. system.

The free parameters of the model are represented by the scales α_c and by the couplings of the underlying chiral Lagrangian, the low-energy constants, that enter the effective potentials $v_{c'c}$. Some of them are usually fixed to already established values, the other are determined in fits to experimental data, mostly to the cross sections at low energies. The $\bar{K}N$ system is specific in providing additional constraints at the threshold energy in a form of available branching ratios and kaonic hydrogen characteristics for the K^- -atomic 1s level energy shift and width caused by the strong interaction. The whole procedure applied in the $\bar{K}N$ as well as in the ηN sectors has been described e.g. in Refs. [ACi4, ACi10, ACi14, ACi16].

When the elementary $\bar{K}N$ (or ηN) system is submerged in the nuclear medium, the MB amplitudes and loop functions become density dependent and one has to consider Pauli blocking and self-energies generated by the interactions of mesons and baryons with the medium. Thus, the propagators $G_c(W)$ and the MB amplitudes $f_{c'c}(W)$ become dependent on the nuclear density ρ , see [ACi7, ACi8] for details. It is worth noting

that unlike many other approaches to coupled channels MB interactions (based on the dimensional regularization of the MB loop function), the separable Prague model provides a natural extension off-the-energy shell due to the g_c form factors. This feature makes it quite suitable for in-medium applications in which the interacting hadrons often need to be treated with the off-shell momenta.

Finally, the chirally motivated MB coupled channels models can be tested when calculating the cross sections (and other observables) of processes involving the rescattering of the MB pair in the final state. A typical example is represented by the $\gamma p \rightarrow K^+ \pi \Sigma$ photoproduction reaction. It was shown in [16] that the amplitude to produce a $\pi \Sigma$ pair with a mass $M_{\pi \Sigma}$ can be calculated as

$$[\mathcal{A}(M_{\pi \Sigma})] = [\mathcal{A}^{\text{tree}}(M_{\pi \Sigma})] + [f(M_{\pi \Sigma})] [\tilde{G}(M_{\pi \Sigma})] [\mathcal{A}^{\text{tree}}(M_{\pi \Sigma})]. \quad (3)$$

Here $\tilde{G}(M_{\pi \Sigma}) = 8\pi M_{\pi \Sigma} G(M_{\pi \Sigma})$ is a diagonal matrix with entries given by suitably regularized MB loop integrals equivalent to those in Eq. (2), $f(M_{\pi \Sigma})$ represents the MB scattering amplitudes (1) and the square brackets mark the vector or matrix character of the involved quantities in the MB channel-space. The amplitudes $\mathcal{A}^{\text{tree}}$ stand for the tree-level MB photoproduction amplitudes constructed within a specific photoproduction model. One should note a close similarity of the Eq. (3)

to a standard Lippmann-Schwinger equation and we present it here only in a simplified form, not showing an explicit dependence on the total c.m. energy (Mandelstam s -variable), projection to a specific partial wave etc, see [16] or [ACi17] for a more complete formulation and further details.

3 Author's contributions to the topic

3.1 $\bar{K}N$ interactions

[ACi13] *Kaonic hydrogen versus the K^-p low energy data*

Following a precise measurement of the 1s level characteristics of the kaonic hydrogen atom by the DEAR collaboration at DAΦNE [17], the paper has presented an exact theoretical solution to the K^- -proton bound-state problem formulated in the momentum space. The kaonic hydrogen characteristics were calculated simultaneously with the available low-energy K^-p cross sections. In the strong-interaction sector the MB interactions were described by means of an effective chirally motivated separable potential and its parameters were fitted to the data. The paper has pointed out a deficiency of the modified Deser-Trueman formula [18] that relates the K^-p scattering length to the K^- -atomic 1s level energy shift and width. It also confirmed the incompatibility of the DEAR results with the other K^-p

reactions data.

[ACi4] *Separable potential model for K^-N interactions at low energies*

Effective separable MB potentials were constructed to match the equivalent chiral amplitudes up to the second order in external meson momenta. The model parameters (low-energy constants) were fitted to the threshold and low-energy K^-p data. In the process, the K^- -proton bound-state problem was solved exactly in the momentum space and the $1s$ level characteristics of the kaonic hydrogen were computed simultaneously with the available K^-p cross sections. The model was also used to describe a general form of the $\pi\Sigma$ mass spectrum and the energy dependence of the K^-n amplitude.

[ACi9] *Chirally motivated $\bar{K}N$ amplitudes for in-medium applications*

After the new precise kaonic hydrogen data were released by the SIDDHARTA collaboration [19], we presented a new fit of our coupled-channel model for the MB interactions. The kaon-nucleon amplitudes generated by the model were found fully consistent with our earlier studies. In the paper, we argued that a sharp increase of the real part of the in-medium K^-p amplitude at subthreshold energies provides a link between the shallow K^- -nuclear optical potentials

obtained microscopically from threshold $\bar{K}N$ interactions and the phenomenological deep ones deduced from kaonic atoms data. The impact on the A -dependence of the Λ -hypernuclear formation rates measured in reactions with stopped kaons was briefly discussed in the paper as well.

[ACi12] *Effective model for in-medium $\bar{K}N$ interactions including the $L = 1$ partial wave*

The coupled channels model of MB interactions based on the effective chiral Lagrangian was extended to account explicitly for the $\Sigma(1385)$ resonance that dominates the p-wave $\bar{K}N$ and $\pi\Sigma$ interactions at energies below the $\bar{K}N$ threshold. The presented model aimed at a uniform treatment of the $\Lambda(1405)$ and $\Sigma(1385)$ dynamics in the nuclear medium. We have demonstrated the applicability of the model by confronting its predictions with the free-space scattering data, and then followed with discussing the impact of nuclear matter on the $\pi\Sigma$ mass spectrum and on the energy dependence of the K^-p branching ratios. The latter is relevant for understanding e.g. the $\pi\Sigma$ formation rates reported in measurements of hypernuclear decays.

[ACi13] *On the pole content of coupled channels chiral approaches used for the $\bar{K}N$ system*

The paper presented a comparative analysis of several theoretical approaches to $\bar{K}N$ interaction at low energies, all

of them based on the chiral SU(3) dynamics and including NLO contributions. A particular attention was paid to a detailed discussion of their pole contents. It was demonstrated that the approaches lead to very different predictions for the K^-p amplitude extrapolated to subthreshold energies as well as for the K^-n amplitude. The origin of the poles generated by the models was traced to the so-called zero coupling limit, in which the inter-channel couplings are switched off. This procedure provided new insights into the pole contents of the considered approaches. In particular, different concepts of forming the $\Lambda(1405)$ resonance were revealed and constraints related to the appearance of such poles in a given approach were discussed. In the isovector sector, the cusp structure observed in the energy dependence of the elastic K^-n amplitude was related to the isovector pole located close to the physical region.

[ACi15] *Importance of chiral constraints for the pole content of the $\bar{K}N$ scattering amplitude*

We have critically examined the $\bar{K}N$ coupled-channel approach presented in [20] and demonstrated that it violates constraints imposed by chiral symmetry of QCD. The origin of this violation was traced back to the off-shell treatment of the chiral-effective vertices, in combination with the use of non-relativistic approximations and the chosen regularization scheme. We have proposed an improved ver-

sion of the approach, which is directly given by a resummation of relativistic Feynman graphs of baryon chiral perturbation theory, and is in accord with the chiral symmetry constraint. Within this improved model, two poles are generated dynamically in the isoscalar $\pi\Sigma - \bar{K}N$ sector, in contrast with the non-relativistic model of [20] in which only one such pole was reported.

[ACi16] *SU(3) flavor symmetry considerations for the $\bar{K}N$ coupled channels system*

In this paper we studied the impact of SU(3) flavor symmetry breaking on the properties of dynamically generated Λ^* states within an effective separable potential model describing the coupled channels $\bar{K}N$ system. The model is based on the chiral MB Lagrangian at next-to-leading order, and constitutes an improvement over the previous version of the Prague model [ACi9], with its applicability being extended to higher energies covering the $\Lambda(1670)$ resonance region. It was demonstrated that the ratios of channel couplings to the resonant states can vary dramatically when the flavor breaking is gradually switched off, tracing a path to the restored SU(3) symmetry. Therefore, we have argued that the couplings determined from physical observables cannot be used to relate reliably a given resonance to a specific flavor multiplet.

3.2 ηN and $\eta' N$ interactions

[ACi10] *Chirally motivated separable potential model for ηN amplitudes*

The ηN interaction was analyzed using a coupled channel separable potential model that implements the chiral symmetry. The model provides ηN scattering length $a_{\eta N} \sim 0.7$ fm and in-medium subthreshold attraction that are most likely sufficient to generate η -nuclear bound states, in contrast with previous predictions by other chirally motivated models that generate much weaker ηN attraction. The energy dependence of the ηN amplitude and pole content of the model were discussed as well. Within the model framework, the baryon resonances $N(1535)$ and $N(1650)$ originate from the same state when the inter-channel couplings are switched off. Although this feature is likely caused by our restriction to two body MB channels and may not be realistic, it is still interesting in itself.

[ACi14] *Coupled channels approach to ηN and $\eta' N$ interactions*

In this paper, we presented a coupled channels separable potential approach to ηN and $\eta' N$ interactions using a chiral-symmetric interaction kernel. The s -wave πN amplitudes and $\pi^- N$ induced total cross sections were reproduced satisfactorily in a broad interval of energies despite limiting

the channel space to two-body interactions of pseudoscalar mesons with the baryon ground-state octet. It was demonstrated that an explicit inclusion of the η_0 meson singlet field leads to a more attractive ηN interaction, with the real part of the scattering length exceeding 1 fm. The $\eta' N$ diagonal coupling appears sufficient to generate an $\eta' N$ bound state when the non-diagonal channel couplings are switched off but the inter-channel dynamics moves the respective pole far from the physical region making the $\eta' N$ interaction repulsive at energies around the channel threshold. The $N(1535)$ and $N(1650)$ resonances are generated dynamically within the model and the origin and properties of the S-matrix poles assigned to them was studied in detail. We have also hinted at a chance that the $N(1895)$ state might be formed dynamically if a suitably varied model setting was found.

3.3 $K^- N$ and ηN systems in nuclear matter

[ACi1] *K^- -nucleus relativistic mean field potential consistent with kaonic atoms*

K^- atomic data were used to test several models of the K^- nucleus interaction. At first, we found that the data could not be reproduced when employing a standard relativistic mean field potential that disregards the $\Lambda(1405)$ dynamics at low densities. The same conclusion was reached for

the $t\rho$ optical potential constructed from chirally motivated K^-N coupled channel amplitudes and incorporating the $\Lambda(1405)$ dynamics. There, the Lippmann-Schwinger Eq. (1) was solved in the nuclear medium, taking into account the Pauli exclusion principle in the intermediate nucleon states, as a function of the Fermi momentum p_F within the Fermi gas model. Fermi motion and nucleon binding effects were also considered, but turned out to play only a secondary role. Finally, the K^- atomic data were reproduced successfully with a hybrid model that combines theoretically motivated RMF approach in the nuclear interior and a completely phenomenological density dependent potential, that respects the low density theorem in the nuclear surface region. This best-fit K^- optical potential was found to be strongly attractive, with a depth around 180 MeV at the nuclear interior, in agreement with previous phenomenological analyses [21]. This is in contrast with the depth of the optical potential constructed from the chirally motivated K^-N amplitudes, $V_{K^-}(\rho = \rho_0) = t(\rho_0)\rho_0$, well below 100 MeV. The paper had an impact on astrophysical models as a deep \bar{K} -nuclear optical potential is a prerequisite for a kaon condensation at high densities, as realized e.g. in neutron stars.

[ACi2] *Study of chirally motivated low-energy K^- optical potentials*

In this work, the K^- optical potential constructed from the in-medium K^-N amplitudes was evaluated self consistently. The parameters of the *MB* coupled channels model were fitted to a select subset of the low-energy data plus the K^- atomic data available throughout the periodic table. The resulting attractive K^- optical potentials are relatively *shallow*, with central depth of the real part about 60 MeV, for a fairly reasonable reproduction of the K^- -atomic data. Relatively *deep* attractive potentials of depth about 180 MeV, that result in phenomenological approaches, provide superior fits but were ruled out within chirally motivated models. In the paper, we argued that a different physical data input may be necessary to distinguish between shallow and deep K^- optical potentials and proposed the (K_{stop}^-, π) reaction as a candidate to provide such a test. The forward (K^-, π) differential cross sections for the production of relatively narrow deeply bound K^- nuclear states were evaluated for deep optical potentials, yielding values considerably lower than those estimated before.

[ACi5] *Λ hypernuclear production in (K_{stop}^-, π) reactions reexamined*

In the paper, we presented DWIA calculations of the Λ hypernuclear production rates in stopped K^- reactions on sev-

eral p -shell targets that were used in experiments by the FINUDA Collaboration [22, 23]. Chirally motivated $K^- N \rightarrow \pi \Lambda$ in-medium transition amplitudes were employed and the sensitivity of the calculated rates to the initial K^- -atomic wave functions and final pion distorted waves was studied. The calculated rates were compared with measured rates, wherever available, confirming earlier observations that (i) the calculated rates are generally lower than the measured rates, and (ii) the deeper the K^- -nuclear potential, the worse is the discrepancy. The A dependence of the calculated production rates was also discussed in the paper and proposed as a useful tool to resolve the topical issue of the depth of the K^- -nuclear potential near threshold.

[ACi6] *Constraints on the threshold K^- nuclear potential from FINUDA ${}^A Z (K_{\text{stop}}^-, \pi^-) {}^A Z$ spectra*

$1s_\Lambda$ hypernuclear formation rates in stopped K^- reactions on several p -shell targets were derived from hypernuclear formation spectra measured by the FINUDA Collaboration [23] and were compared with calculated $1s_\Lambda$ formation rates based on a chirally motivated coupled channel model presented in [ACi4]. In contrast with previous calculations, within our model the calculated rates depend weakly on the depth of the threshold K^- -nuclear potential. The A dependence of the calculated $1s_\Lambda$ rates was found in fair agreement with that of the experimental $1s_\Lambda$ rates, show-

ing a slight preference for a deep density dependent potential with $\text{Re } V_{K^-}(\rho_0) \sim -(150-200)$ MeV, but not excluding shallow potentials providing $\text{Re } V_{K^-}(\rho_0) \sim -50$ MeV and advocated by other authors. The observations made in the paper originate from a substantial energy and density dependence found for the in-medium subthreshold $K^-n \rightarrow \pi^- \Lambda$ branching ratio that enters the hypernuclear formation rate calculations. Unfortunately, the calculated rates were found too small, about 15% of the derived experimental rates, most likely due to difficulties inherent in normalisation of the latter.

[ACi7] *Chirally motivated K^- nuclear potentials*

In this work, we argued that the K^-N scattering amplitudes need to be evaluated at subthreshold energies when the K^- -nuclear optical potential $V_{K^-}^{t\rho} = t_{K^-N} \rho$ is constructed. The downward energy shift was related to the impulse approximation in which the many-body K^-N amplitude evaluated at the K^- -nuclear energy is replaced by the two-body amplitude at its c.m. energy. The energy of the MB system is then determined self consistently when confronting the kaonic atoms data across the periodic table. As a result, considerably deeper K^- -nuclear potentials are obtained if compared with the shallow potentials derived in other approaches based on threshold K^-N amplitudes. When we incorporated phenomenologically the K^-NN contributions,

a very deep optical potential potential was obtained with $\text{Re } V_{K^-}^{t\rho+phen.} \sim -180 \text{ MeV}$, in agreement with density dependent potentials obtained in purely phenomenological fits to the data.

The results presented in our paper provided for the first time a microscopic link between shallow K^- nuclear potentials [ACi2, 5, 24] obtained from threshold K^-N interactions and phenomenological deep ones deduced from kaonic atom data [21, 25]. Self consistent dynamical calculations of K^- -nuclear quasibound states generated by the $V_{K^-}^{t\rho}$ potential were also reported and discussed in the letter.

[ACi8] *K^- nuclear potentials from in-medium chirally motivated models*

The paper builds on a previous letter [ACi7] providing more details on the adopted formalism and presenting some additional results with a more detailed discussion on the analysis of kaonic atoms data and the predictions of the K^- quasibound nuclear states. In particular, an impact of the chiral Lagrangian NLO terms on the K^-N amplitudes and the in-medium dynamics of the coupled-channel MB model were discussed as well as an addition of the complex ρ - and ρ^2 -dependent phenomenological terms to the K^- -nuclear optical potential. We have demonstrated that the ρ^2 contributions dominate the potential, most likely representing the

$\bar{K}NN \rightarrow YN$ absorption and dispersion, outside the scope of the MB chiral models. We also looked at the effects of p-wave interactions generated by the $\Sigma(1385)$ subthreshold resonance and found them secondary to those of the s-wave contributions dominated by the $\Lambda(1405)$ resonance.

[ACi11] *In-medium ηN interactions and η -nuclear bound states*

The in-medium ηN interaction near and below threshold was constructed from a free-space coupled channels model presented in [ACi10] that captures the physics of the $N(1535)$ resonance. Nucleon Pauli blocking and hadron self-energies were accounted for in a similar manner as in the $\bar{K}N$ sector [ACi7, ACi8]. The resulting energy-dependent in-medium interaction was used in self-consistent dynamical calculations of η -nuclear bound states. Narrow states of width $\Gamma_\eta \sim 2$ MeV were found across the periodic table, beginning from $A \sim 10$. The binding energy of the $1s_\eta$ state increases with A , reaching a value of $B_{1s}(\eta) \sim 15$ MeV. The implications of our self-consistency procedure were discussed with respect to approaches adopted in other works.

3.4 $\pi\Sigma$ photoproduction

[ACi17] *Testing chiral unitary models for the $\Lambda(1405)$ in $K^+\pi\Sigma$ photoproduction*

We have adopted a novel formalism for the low-energy anal-

ysis of the $\gamma p \rightarrow K^+ \pi \Sigma$ photoproduction reaction, outlined recently in [16], to calculate the $\pi \Sigma$ invariant mass distributions in the $\Lambda(1405)$ resonance region. The approach adheres to constraints arising from unitarity, gauge invariance and chiral perturbation theory, and was used without adjusting any model parameters. It was found that the MB rescattering in the final state has a major impact on the magnitude and structure of the generated spectra that were compared with the experimental data from the CLAS collaboration [26]. In the paper we demonstrated a large sensitivity of the theoretical predictions to the choice of the coupled-channel $\pi \Sigma - \bar{K} N$ model amplitudes which should enable one to constrain the parameter space of these models.

[ACi18] *Constraining the chirally motivated $\pi \Sigma - \bar{K} N$ models with the $\pi \Sigma$ photoproduction mass spectra*

The paper presented a first time attempt on a combined fit of the $K^- p$ low-energy data and the $\pi \Sigma$ photoproduction mass spectra, performed without fixing the MB rescattering amplitudes to a specific $\pi \Sigma - \bar{K} N$ coupled channels model obtained from fitting exclusively the $K^- p$ data. The formalism adopted to describe the photoproduction process was based on chiral perturbation theory and employs a limited number of free parameters. The achieved reproduction of the photoproduction mass distributions was not quite

satisfactory (in particular for the $\pi^-\Sigma^+$ channel), leaving a room for improving the photo-kernel construction. However, it still provides additional constraints on the positions of the $\Lambda(1405)$ poles. In particular, the presented models tend to limit the mass of the lower pole and yield a larger width of the $\bar{K}N$ related pole at a higher mass.

4 Bibliography

References to the presented results

- ACi1 E. FRIEDMAN, A. GAL, J. MAREŠ, and A. CIEPLÝ:
' K^- -nucleus relativistic mean field potential consistent with kaonic atoms',
[Phys. Rev. C **60**, 024314 \(1999\)](#).
- ACi2 A. CIEPLÝ, E. FRIEDMAN, A. GAL, and J. MAREŠ:
'Study of chirally motivated low-energy K^- optical potentials',
[Nucl. Phys. A **696**, 173–193 \(2001\)](#).
- ACi3 A. CIEPLÝ and J. SMEJKAL:
'Kaonic hydrogen versus the $K^- p$ low energy data',
[Eur. Phys. J. A **34**, 237–241 \(2007\)](#).
- ACi4 A. CIEPLÝ and J. SMEJKAL:
'Separable potential model for $K^- N$ interactions at low energies',
[Eur. Phys. J. A **43**, 191–208 \(2010\)](#).
- ACi5 V. KREJČÍŘÍK, A. CIEPLÝ, and A. GAL:
' Λ hypernuclear production in (K_{stop}^-, π) reactions reexamined',
[Phys. Rev. C **82**, 024609 \(2010\)](#).
- ACi6 A. CIEPLÝ, E. FRIEDMAN, A. GAL, and V. KREJČÍŘÍK:
'Constraints on the threshold K^- nuclear potential from FINUDA
 ${}^A Z (K_{\text{stop}}^-, \pi^-) {}^A Z$ spectra',
[Phys. Lett. B **698**, 226–230 \(2011\)](#).
- ACi7 A. CIEPLÝ, E. FRIEDMAN, A. GAL, D. GAZDA, and J. MAREŠ:
'Chirally motivated K^- nuclear potentials',
[Phys. Lett. B **702**, 402–407 \(2011\)](#).

- ACi8 A. CIEPLÝ, E. FRIEDMAN, A. GAL, D. GAZDA, and J. MAREŠ:
' K^- nuclear potentials from in-medium chirally motivated models',
[Phys. Rev. C **84**, 045206 \(2011\)](#).
- ACi9 A. CIEPLÝ and J. SMEJKAL:
'Chirally motivated $\bar{K}N$ amplitudes for in-medium applications',
[Nucl. Phys. A **881**, edited by A. Gal, O. Hashimoto, and J. Pochodzalla, 115–126 \(2012\)](#).
- ACi10 A. CIEPLÝ and J. SMEJKAL:
'Chirally motivated separable potential model for ηN amplitudes',
[Nucl. Phys. A **919**, 46–66 \(2013\)](#).
- ACi11 A. CIEPLÝ, E. FRIEDMAN, A. GAL, and J. MAREŠ:
'In-medium ηN interactions and η nuclear bound states',
[Nucl. Phys. A **925**, 126–140 \(2014\)](#).
- ACi12 A. CIEPLÝ and V. KREJČÍŘÍK:
'Effective model for in-medium $\bar{K}N$ interactions including the $L = 1$ partial wave',
[Nucl. Phys. A **940**, 311–330 \(2015\)](#).
- ACi13 A. CIEPLÝ, M. MAI, U.-G. MEIBNER, and J. SMEJKAL:
'On the pole content of coupled channels chiral approaches used for the $\bar{K}N$ system',
[Nucl. Phys. A **954**, 17–40 \(2016\)](#).
- ACi14 P. C. BRUNS and A. CIEPLÝ:
'Coupled channels approach to ηN and $\eta' N$ interactions',
[Nucl. Phys. A **992**, 121630 \(2019\)](#).
- ACi15 P. C. BRUNS and A. CIEPLÝ:
'Importance of chiral constraints for the pole content of the $\bar{K}N$ scattering amplitude',
[Nucl. Phys. A **996**, 121702 \(2020\)](#).

- ACi16 P. C. BRUNS and A. CIEPLÝ:
'SU(3) flavor symmetry considerations for the $\bar{K}N$ coupled channels system',
[Nucl. Phys. A **1019**, 122378 \(2022\)](#).
- ACi17 P. C. BRUNS, A. CIEPLÝ, and M. MAI:
'Testing chiral unitary models for the $\Lambda(1405)$ in $K^+\pi\Sigma$ photoproduction',
[Phys. Rev. D **106**, 074017 \(2022\)](#).
- ACi18 A. CIEPLÝ and P. C. BRUNS:
'Constraining the chirally motivated $\pi\Sigma - \bar{K}N$ models with the $\pi\Sigma$ photoproduction mass spectra',
[Nucl. Phys. A **1043**, 122819 \(2024\)](#).

Other references

- 1 S. Weinberg, 'Phenomenological Lagrangians', [Physica A **96**, edited by S. Deser, 327–340 \(1979\)](#).
- 2 J. Gasser and H. Leutwyler, 'Chiral Perturbation Theory: Expansions in the Mass of the Strange Quark', [Nucl. Phys. B **250**, 465–516 \(1985\)](#).
- 3 J. Gasser, M. E. Sainio, and A. Svarc, 'Nucleons with chiral loops', [Nucl. Phys. B **307**, 779–853 \(1988\)](#).
- 4 N. Kaiser, P. B. Siegel, and W. Weise, 'Chiral dynamics and the low-energy kaon - nucleon interaction', [Nucl. Phys. A **594**, 325–345 \(1995\)](#).
- 5 E. Oset and A. Ramos, 'Nonperturbative chiral approach to S-wave $\bar{K}N$ interactions', [Nucl. Phys. A **635**, 99–120 \(1998\)](#).

- 6 J. A. Oller and U. G. Meißner, ‘Chiral dynamics in the presence of bound states: Kaon nucleon interactions revisited’, *Phys. Lett. B* **500**, 263–272 (2001).
- 7 D. Jido, J. A. Oller, E. Oset, A. Ramos, and U. G. Meißner, ‘Chiral dynamics of the two $\Lambda(1405)$ states’, *Nucl. Phys. A* **725**, 181–200 (2003).
- 8 T. Hyodo and D. Jido, ‘The nature of the $\Lambda(1405)$ resonance in chiral dynamics’, *Prog. Part. Nucl. Phys.* **67**, 55–98 (2012).
- 9 M. Mai, ‘Review of the $\Lambda(1405)$: A curious case of a strangeness resonance’, *Eur. Phys. J. ST* **230**, 1593–1607 (2021).
- 10 N. Kaiser, P. B. Siegel, and W. Weise, ‘Chiral dynamics and the $S_{11}(1535)$ nucleon resonance’, *Phys. Lett. B* **362**, 23–28 (1995).
- 11 T. Inoue, E. Oset, and M. J. Vicente Vacas, ‘Chiral unitary approach to S wave meson baryon scattering in the strangeness $S = 0$ sector’, *Phys. Rev. C* **65**, 035204 (2002).
- 12 E. Friedman and A. Gal, ‘ K^-N amplitudes below threshold constrained by multinucleon absorption’, *Nucl. Phys. A* **959**, 66–82 (2017).
- 13 A. Gal, E. Friedman, N. Barnea, A. Cieplý, J. Mareš, and D. Gazda, ‘In-medium \bar{K}^- and η -meson interactions and bound states’, *Acta Phys. Polon. B* **45**, 673 (2014).
- 14 J. Hrtánková and J. Mareš, ‘Are there any narrow K^- - nuclear states?’, *Phys. Lett. B* **770**, 342–347 (2017).
- 15 A. Gal, E. V. Hungerford, and D. J. Millener, ‘Strangeness in nuclear physics’, *Rev. Mod. Phys.* **88**, 035004 (2016).
- 16 P. C. Bruns, ‘A formalism for the study of $K^+\pi\Sigma$ photoproduction in the $\Lambda^*(1405)$ region’, e-Print: 2012.11298 [nucl-th] (2020).

- 17 G. Beer et al. (DEAR), ‘Measurement of the kaonic hydrogen X-ray spectrum’, *Phys. Rev. Lett.* **94**, 212302 (2005).
- 18 U.-G. Meißner, U. Raha, and A. Rusetsky, ‘Spectrum and decays of kaonic hydrogen’, *Eur. Phys. J. C* **35**, 349–357 (2004).
- 19 M. Bazzi et al. (SIDDHARTA), ‘A new measurement of kaonic hydrogen X-rays’, *Phys. Lett. B* **704**, 113–117 (2011).
- 20 J. Révai, ‘Are the chiral based $\bar{K}N$ potentials really energy dependent?’, *Few Body Syst.* **59**, 49 (2018).
- 21 C. J. Batty, E. Friedman, and A. Gal, ‘Strong interaction physics from hadronic atoms’, *Phys. Rept.* **287**, 385–445 (1997).
- 22 M. Agnello et al. (FINUDA), ‘First results on ${}_{\Lambda}^{12}\text{C}$ production at DAΦNE’, *Phys. Lett. B* **622**, 35–44 (2005).
- 23 M. Agnello et al. (FINUDA), ‘Hypernuclear spectroscopy with K^- at rest on ${}^7\text{Li}$, ${}^9\text{Be}$, ${}^{13}\text{C}$ and ${}^{16}\text{O}$ ’, *Phys. Lett. B* **698**, 219–225 (2011).
- 24 A. Baca, C. Garcia-Recio, and J. Nieves, ‘Deeply bound levels in kaonic atoms’, *Nucl. Phys. A* **673**, 335–353 (2000).
- 25 J. Mareš, E. Friedman, and A. Gal, ‘ \bar{K} -nuclear bound states in a dynamical model’, *Nucl. Phys. A* **770**, 84–105 (2006).
- 26 K. Moriya et al. (CLAS), ‘Measurement of the $\Sigma\pi$ photoproduction line shapes near the $\Lambda(1405)$ ’, *Phys. Rev. C* **87**, 035206 (2013).